Calculus 251:C3 Reading Guide - 7/14/2020

Section 17.3 Divergence Theorem

Well, this is it. You have made it to the last section of the textbook! This section is about the Divergence Theorem, the last of the Fundamental Theorems of vector calculus. It establishes the equivalence of the flux integral of a vector field through a closed surface with the triple integral of the divergence of the vector field over the enclosed region. If you think about divergence in terms of sources and sinks, then you could say that divergence at a point measures how much the field represents a flow into or out of that point. Then what the Divergence Theorem is really saying is that if you add up all of the sources and sinks in a region, that tells you what the net flow across the boundary of the region has to be.

Find the following definitions/concepts/formulas/theorems:

- Theorem: Divergence Theorem

Yes, that's really the only thing you need to find. The rest of the section (including its other two theorems) consists of examples and explanations. The other two theorems (Flux of the Inverse-Square Field and Uniformly Charged Sphere) are both very useful. You should probably write them down, and you will definitely see them in physics and engineering classes.

As with the first two major theorems in this chapter, the textbook proves a special case of the Divergence Theorem. They only prove the theorem for a box, but you can write any region as the limit of some sum of very small boxes as the size of the boxes tends to zero. So other than some technical details of that limit, this is really what the proof of the theorem looks like in the general case.

Example 1 verifies the Divergence Theorem by computing the integrals on both sides of the theorem directly. Example 2 uses the Divergence Theorem to calculate the flux out of a box. This is defintely less work than integrating across each of the six sides of the box. In example 3, the divergence is zero, so this ugly-looking integral (I don't think they even bothered telling us what the surface is) becomes trivial.

The Graphical Insight after example 3 is a more formal presentation of the "sources and sinks" idea that I mentioned in the opening paragraph.

The subsection on Electrostatics is important. You should read through it including both examples and both theorems. And then you should look at it again sometime between the end of this course and the start of the fall semester. Again, physics is the main reason this chapter exists. Read through the Historical Perspective on p. 1058 for evidence supporting that claim.

Definitely read through the Conceptual Insight and summary after the last theorem. And then you're done!