Calculus 251:C3 Reading Guide - 7/13/2020

Section 17.2 Stokes' Theorem

This section focuses on Stokes' Theorem, a generalization of Green's Theorem to $\mathbb{R}^3$. It establishes the equivalence of the flux integral of the curl of a vector field through a surface with the vector line integral of the field along the boundary of the surface. A very important specific case is that for a *closed* surface, the flux integral of the curl will always be 0 (as long as $\vec{F}$ is reasonably well-behaved).

Find the following definitions/concepts/formulas/theorems:

- closed surface
- boundary orientation (for the boundary of a surface, not a plane region as in §17.1)
- Theorem: Stokes' Theorem
- vector potential

The paragraphs before the theorem define the boundary of a surface and describe how we orient those boundaries. This is essential information for the rest of the section. As with Green's Theorem, the textbook proves a special case of Stokes' Theorem. The special case is the surface is a function $z = f(x, y)$, and they even leave out some of the details of that case. You should at least scan the proof.

Example 1 verifies Stokes' Theorem for a combination of $\vec{F}$ and $\mathcal{S}$ by computing the integrals on both sides of the theorem directly. Example 2 uses Stokes' Theorem (and a neat trick that only works because $\mathcal{S}$ is a plane) to show that a vector line integral around a triangle is 0. I'm not entirely sure that this was less work than the three line integrals would have been. But in many cases, applying the theorem will be easier that whatever the other side of the equation would have been. It's always good to have options.

Example 3 and the conceptual insight preceding it involve vector potentials. When we found the potential of a vector field $\vec{F}$, we were looking for a scalar function whose gradient was $\vec{F}$. A vector potential for $\vec{F}$ is another vector field whose *curl* is $\vec{F}$.

Example 4 is an application of Stokes' Theorem to a magnetic field problem. Definitely worth a read, if only as a reminder of how much physics is the motivation for much of vector calculus.

I suggest reading the conceptual insights before and after Example 4. They may help you, they may not. But if there is something in one or both of them that resonates with you, it will help build your intuition for what is happening in the examples.