Calculus 251:C3 Reading Guide - 7/6/2020

Section 16.5 Surface Integrals of Vector Fields

In this section, we explore the second type of surface integral, the vector surface integral. We generally think about these integrals as the flow of something from one side of our surface to the other. We could be taking about water flowing through a membrane, or electric flux, or the flow of heat through some surface. One important point is that because we are talking about flow through a surface, we are interested in the normal component of the vector field. The tangential component is moving along the surface at the point of tangency, so does not get counted as we are computing flow across the surface. Another point is that we need our surface to be oriented, because we need to know which direction through the surface will be counted as positive flux and which direction will be counted as negative flux.

Find the following definitions/concepts/formulas/theorems:

- orientation (for a parametrized surface)
- normal component (of a vector field w.r.t. a surface)
- vector surface integral
- flux (of a field through a surface)
- Theorem: Vector Surface Integral
- vector surface differential
- flow rate (definition and formula for flow rate through a surface)

All of the discussion leading up the the Theorem is worth reading. You will also hear me describe what is going on in lecture, but seeing these concepts more than once is helpful. This is the setup for what vector surface integrals mean and how we compute them.

Examples 1, 2, and 3 all follow the same pattern. Parametrize the surface (if the parametrization wasn't given), calculate the tangent and normal vectors, find the normal component of the vector field, integrate over the parameter domain.

The conceptual insight after example 3 discusses the Möbius strip and why it is a non-orientable surface. The basic idea is that you can't talk about the flow from one side to the other, because the surface only has one side. Cool stuff.

Definitely read through the subsections on fluid flux and electric/magnetic fields. The formulas are really just notation changes particular to the situation. Faraday's Law of Induction is often stated as a partial differential equation, but the integral form presented here is equivalent.

One other nice feature of this section is the list "Types of Integrals" at the end. It is a summary of the types of integrals discussed in the entire chapter. We only briefly mentioned the third one, a vector line integral to calculate flux across a curve, but we will spend more time on it this week (both in §17.1 and when we have our review on Wednesday).