Calculus 251:C3   Reading Guide - 7/2/2020

Section 16.4 Parametrized Surfaces and Surface Integrals

When we computed line integrals, we needed parametrized curves along which to integrate. This section is a natural extension of that concept. We will now be parametrizing surfaces and considering integrals over those surfaces. For a curve, we only needed one parameter because a curve is essentially one dimensional (i.e. you can only move forwards or backwards along it) even though the curve itself may have been in 2 or 3 dimensional space. We really treated a curve as a deformed line segment, and worked out how to integrate along it using single-variable calculus with respect to the parameter.

We can think about a surface as a deformed region of a plane, parametrize it using two parameters (because it is essentially a two dimensional object even though it resides in 3 or more dimensions), and use a double integral with respect to those two parameters. In this section, we will only be integrating scalar functions over surfaces (the analogue of scalar line integrals). In the next section, we will consider surface integrals of vector fields.

Find the following definitions/concepts/formulas/theorems:

- parametrized surface
- parameter domain
- Parametrization formulas for cylinders, spheres, and graphs
- grid curves
- regular parametrization (for surfaces - same general idea as for curves, but different details)
- normal vector (same old idea - new context)
- surface area formula (the double integral version)
- surface integral
- Theorem: Surface Integrals and Surface Area
- surface area differential
- equations $\boxed{8}$ and $\boxed{9}$ - very useful shortcuts for surface integrals over graphs

Examples 1, 2, 3, and 4 are parametrizations of various surfaces. You really need to make sure you understand these, because nothing else in this section or the next will make sense if you don't.

Read through the bit about grid curves, normal vectors, and tangent planes. These are the pieces we need to put together in order to compute surface integrals. Examples 5 and 6 illustrate how to describe these objects.

The subsection on surface area should remind you of §15.6 Change of Variables. The idea is very similar, in that we are looking at mapping from a plane to a surface, and trying to figure out how the area of a small rectangle scales across the mapping.

Examples 7, 8, and 9 are (finally) compuations of surface integrals. All of the other concepts in this section are assembled here. Notice that if you integrate 1 dS you are computing a surface area, and if you integrate some other scalar function you are computing something else. Example 7 does both of these, so you can see them in action on the same parametrized surface.

You are welcome to read or skip the section on Gravitational Potential of a Sphere. It's cool, but you may have enough other things to think about. If you are a true alpha nerd, you may well read this section and think about gravity calculations inside the Death Star. Just saying...