**Calculus 251:C3     Reading Guide - 7/1/2020**

**Section 16.3 Conservative Vector Fields**

This section explores conservative vector fields in some depth. It's not that other vector fields aren't interesting, it's just that many of the vector fields we see in physics are conservative. Gravitational fields and electrostatic fields in particular are conservative, which is one of the reasons why we study conservative vector fields in such detail. As you will see, conservative vector fields have some very nice properties which will make much of our analysis easier.

Notational convention: $\oint_{\mathcal{C}} \vec{F} \cdot d\vec{r}$ and $\int_{\mathcal{C}} \vec{F} \cdot d\vec{r}$ really mean the same thing. The circle on the integral sign is just a way to indicate that we are integrating on a *closed* curve.

Find the following definitions/concepts/formulas/theorems:

- circulation
- path independence
- Theorem: Fundamental Theorem for Conservative Vector Fields
- equipotential curves (really just level curves of a potential)
- Theorem: Path independence iff conservative
- potential/kinetic/total energy
- Theorem: Conservation of Energy
- simply connected (for a region in $\mathbb{R}^2$)
- Theorem: Existence of a Potential Function

You should definitely read the proofs of the first two theorems. They're not bad, and they link the new results in this chapter to previous results that we have used quite a bit. Neither one of them should be that hard to understand.

Examples 1, 2, and 3 are calculations that are *much* simpler than the line integrals from the previous section. The reason they are simpler is because we know the vector fields are conservative. Any time you are trying to compute a vector line integral, you should ask yourself, "Is my vector field conservative?" If the answer is yes, you will end up doing a lot less work to get the same result. You should definitely make sure you understand these. The conceptual insight after example 3 is certainly worth a quick read.

The subsection on Conservative Fields in Physics is the set of motivating examples for the existence of vector calculus and the study of conservative vector fields. Examples 4 and 5 are typical examples of physics problems involving a gravitational field and and electric field.

Examples 6 and 7 show the procedure for finding a potential for a conservative vector field.

Note that we did discuss this back in §14.5 when we defined the gradient.

The Assumptions Matter section and example 8 show what can go wrong if the domain of our vector field is not simply connected. The vortex field does satisfy $\mathrm{curl}(\vec{F}) = 0$, but it is not conservative. The rest of the section discusses what happened in this example, and gives you a few ways to think about it. Worth a read, but don't stress too much about it if it doesn't completely make sense on the first try.