Calculus 251:C3 Reading Guide - 6/30/2020

Section 16.2 Line Integrals

This section is all about line integrals. As the book notes, it might be appropriate to call these "curve integrals" or "path integrals" instead. But the same was true of contour lines, which might more accurately be called contour curves. There are two types, scalar line integrals and vector line integrals, the difference being the type of function you are integrating. The general intuition is that you are moving along some path in $\mathbb{R}^3$ (or $\mathbb{R}^2$) and you are adding up the function values along that path. In the case of a scalar line integral, you are adding up the function values (so very much like an integral from Calc II, except that you are summing along a path instead of just an interval). In the case of a vector field, you are summing the tangential components along the path. The way to think about this is that at each point (P) on the path you are only really interested in the projection of the value of $\vec{F}(P)$ onto the direction vector, because that is the part of the vector field in the direction you are moving.

Find the following definitions/concepts/formulas/theorems:

- line integral
- scalar line integral
- Theorem: Computing a Scalar Line Integral
- arc length differential (a/k/a line element)
- formula for arc length differential
- orientation/oriented curve (and positive/negative in this context)
- tangential component
- vector line integral
- vector differential
- formula for vector differential
- Theorem: Computing a Vector Line Integral
- vortex field
- Theorem: Properties of Vector Line Integrals
- flux across a curve (somewhat interesting in its own right, but flux across a surface is the central concept in §16.5, so this is your chance to think about it in two dimensions).

The discussion of scalar line integrals leading up to the first theorem is really an adaptation of Riemann sums to the new context. It's an interesting read and may help develop your intuition for how and why line integrals work. Not absolutely essential, but probably a good idea to spend a little time on it.

Examples 1 and 2 are basic examples of scalar line integrals. You should definitely make sure you understand these.

Examples 3 and 4 are applications of scalar line integrals. These are important. I definitely think it would be fair for me to ask a question about computing the mass of a wire from a mass density function on a quiz or exam. The electric potential example is very important for physics and several branches of engineering, so you should certainly try to work though it.

The discussion of vector line integrals leading up to the second theorem is important. You certainly need to have a good handle on orientation and tangential components. This subsection also mentions the concept of a vector line integral as work, which is one of the most frequent applications of a vector line integral. Hopefully, it makes some sense that if you have a vector field which somehow indicates forces acting on a particle, then summing up the tangential components of those forces as the particle moves along the path should represent the work performed against those forces.

Examples 5, 6, and 7 and the discussions between them are all about how we compute vector line integrals, what they mean, and how we derive some of the properties that will simplify some of our calculations. All important for you to try and wrestle with before class.

Examples 8 and 9 are applications of vector line integrals. Here we have the answer to the time-honored question, "When are we ever going to use this?" In some ways, the reason to work through the applications is so that you have an appreciation for why mathematicians developed these techniques. Almost all of the development of vector calculus was motivated by physics, which is why most of the examples you will see in this chapter are physics examples.