Calculus 251:C3 Reading Guide - 6/24/2020

Section 15.6 Change of Variables

This section discusses how to change variables in a multiple integral. Among other things, this section shows us *why* the integral formulas in §15.4 contained the extra factor of r (or $\rho^2 \sin(\phi)$ for spherical).

Find the following definitions/concepts/formulas/theorems:

- mapping (a/k/a map)
- polar coordinates map
- linear (as it refers to a mapping)
- Jacobian (a/k/a Jacobian determinant - you should notice that there are several notations)
- Theorem: Jacobian of a Linear Map
- approximation formula for area under a non-linear map
- one-to-one map (means the same thing it did in high school algebra)
- C^1 map
- Theorem: Change of Variables Formula
- change of variables formula for three variables

Examples 1, 2, and 3 are the basic examples of how to apply a mapping from $\mathbb{R}^2 \to \mathbb{R}^2$. You need to make sure you understand these before continuing on in the section.

Example 4 is just how to compute the Jacobian of a map, without any context for why it is useful.

The proofs of Theorem 1 (and the conceptual insight that follows it) and Theorem 2 are interesting, but not essential for understanding how to solve problems. You only need to read these if you're curious or if you want to know why these things work.

Examples 5, 6, and 7 are the heart of the matter. Example 5 explains the r in polar and cylindrical integration. Examples 6 and 7 are how you use change of variables to compute integrals. You certainly need to know how all of this works after the lecture. Try to get through as much as you can before.

Example 8 uses an inverse map instead of the forward map. This is a very useful technique, but you may want to wait until after the lecture to tackle it unless you are very comfortable with everything before this.

You should read the bit about change of variables in three dimensions. If you want to know why cylindrical and spherical integration work the way they do, you can derive those formulas using Change of Variables (equation $\boxed{16}$ in this section).