Calculus 251:C3 Reading Guide - 6/23/2020

Section 15.4 Integration in Polar, Cylindrical and Spherical Coordinates

This section discusses double integrals where the domain is expressed in polar coordinates and triple integrals in both cylindrical and spherical coordinates. Again, the motivation is that many situations have a natural symmetry to them which make non-rectangular coordinate systems more convenient. We may get a simpler integrand, less complicated limits of integration, or both.

One very important thing to notice is that there are extra factors which show up in the integrand when you change coordinate systems. So not only do you need to rewrite the function using the techniques we have already discussed, you also get an extra r (for polar or cylindrical) or $\rho^2 \sin\phi$ (for spherical). We will discuss the reasons for that in the lecture after this one (§15.6 Change of Variables). If you can't wait, example 5 in §15.6 discusses the r for polar coordinates. For now, make sure you notice that those extra factors are there and that you include them.

Find the following definitions/concepts/formulas/theorems:

- polar rectangle
- Theorem: Double Integral in Polar Coordinates
- radially simple
- axial symmetry
- Theorem: Triple Integrals in Cylindrical Coordinates
- spherical wedge
- Theorem: Triple Integrals in Spherical Coordinates
- centrally simple

I don't really have a ton to say to guide you through this section. Each of the three integrals (polar, cylindrical, and spherical) has its own subsection. The subsections all give you a description of the theory behind the integral (including construction of the Riemann sums for pol/sph) which includes *when you can use it.* Then there are two examples of the integral and the subsection is over. Do your best to work through as many of the examples as you can.

As with many of the multiple integrals we have already seen, the hard part is setting up the integral and figuring out which of the two or six possible orders of integration will be cleanest. Much of the rest is just slogging through computations, so it is tedious but still important to get right.