Calculus 251:C3 Reading Guide - 6/22/2020

Section 12.7 Cylindrical and Spherical Coordinates

In the plane, we have seen rectangular and polar coordinates. This section extends the idea of polar coordinates to $\mathbb{R}^3$. Why would we want to do such a thing? Well, remember that the area of a cardioid is easy to compute in polar coordinates and a massive pain to do in rectangular. Sometimes different coordinate systems are more convenient because of the inherent symmetry of a situation. So here we are.

Cylindrical coordinates are a very simple generalization of polar coordinates. If you want to know the cylindrical coordinates of a point, you just convert its x and y coordinates to r and θ and leave z alone. One important note here: r is *not* the distance from the origin in cylindrical coordinates, it is the distance from the z-axis.

Spherical coordinates are a little bit more complicated, but can be very powerful. In spherical coordinates, ρ is really just the distance from the origin. θ is still about rotation in the xy-plane: how far do you have to rotate from the positive x-axis towards the positive y-axis to get to your point? ϕ is a measure of how much you have to rotate down from the positive z-axis to get to your point. This means that the positive z-axis has $\phi = 0$, points above the xy-plane have $\phi < \pi/2$, points on the xy-plane have $\phi = \pi/2$, points below the xy-plane have $\phi > \pi/2$, and points on the negative z-axis have $\phi = \pi$. We will always have $\rho > 0$ and $\phi \in [0, \pi]$.

Find the following definitions/concepts/formulas:

- formulas for rectangular↔cylindrical coordinates
- level surfaces (for coordinate systems)
- angle of declination
- formulas for rectangular↔spherical coordinates

Examples 1, 2, and 3 are the basic examples of rectangular↔cylindrical. Example 4 is a nice way to think about how to graph directly in cylindrical coordinates, and a glimpse into why they might be useful.

Examples 5, 6, and 7 are the basic examples of rectangular↔spherical. Examples 8 and 9 are supposed to give you a feel for spherical coordinates, and a glimpse into why they might be useful.

Please make sure the the summary on page 732 (preferably including the pictures) makes it into your formula sheet for quizzes and exams.