Calculus 251:C3 Reading Guide - 6/18/2020

Section 15.3 Triple Integrals

Again, we have a section which is mostly examples. We are going to look at integrals of the form $\iiint_{\mathcal{W}} f(x,y,z)\ dV$ where $\mathcal{W}$ is some region in $\mathbb{R}^3$. There is not too much difference between double and triple integrals, but you do have more choices and additional calculations.

Just to provide some motivation, we will use triple integrals for a few useful applications. In this section, we will see that we can use a triple integral to find the average value of f on $\mathcal{W}$. In §15.5, we will use triple integrals to calculate mass given a density function, find the center of mass of an object, and find moments of inertia. Triple integrals are also used in calculations of total charge, heat content, and total energy.

Find the following definitions/concepts/formulas/theorems:

- integrable (same idea as definition from §15.1)
- Theorem: Fubini's Theorem for Triple Integrals
- number of orders in which you can write a triple integral
- x-simple, y-simple, z-simple
- projection (onto a coordinate plane)
- Theorem: interated integrals
- volume as a triple integral
- hypervolume (mostly because I want you to know I didn't make the word up)

Example 1 is the most basic of all triple integrals - integration over a box.

Examples 2, 3, 4, and 5 are most of the section. You should try to work through all of them. There are some figures to guide you, but these examples are all quite a bit of work to get through. Do your best.

Example 6 is an average temperature calculation, and a much simpler integral than several of the previous examples. Note that they refer to the Mean Value Theorem at the end of the example, but I don't think the book ever states the triple integral version. It is really no different than MVT for one or two variables, though.

The discussion at the end of the section about the volume of spheres in higher dimensions is fascinating. It is also where you will find the definiton of hypervolume. But if you are really tired when you get this far, please don't force yourself to read this part. Go back and look at it the week after the final exam if you're curious.