Calculus 251:C3 Reading Guide - 6/17/2020

Section 15.2 Double Integrals over More General Regions

The bad news is this is a very long section. The good news is that it consists mostly of examples. The reason there are so many examples is because the domains over which we are integrating can be many different shapes in RR^2. The main idea is that you want to set up an iterated integral as we did for rectangles, but the limits of integration for the inner integral will usually be in terms of the other variable. Honestly, you jet need to look at lots of examples to see how that works in practice.

Find the following definitions/concepts/formulas/theorems:

- piecewise smooth
- integrable (slight addition to definition from §15.1)
- Theorem: Existence of the integral on $\mathcal{D}$ with piecewise smooth boundary
- Formula for estimating area of $\mathcal{D}$ with Riemann sums
- horizontally/vertically simple
- Theorem: double integrals for horizontally/vertically simple domains
- Theorem: bounding integrals
- average value (a/k/a mean value)
- connected (for a domain)
- Theorem: Mean Value Theorem for Double Integrals
- decomposing a domain into smaller domains (the idea is much more important than the formulas)

The first couple of pages deal with how we generalize domains and what properties they need to have. Not critically important to understanding what follows, but you should take a quick look to get a sense of why we can use the techniques from the previous section.

Example 1 is about estimating a double integral with a finite Riemann sum. Not something we will generally do by hand, but this is pretty much how computerized numerical integration works. At least take a quick look, without worrying about the arithmetic.

The proof of Theorem 2 states that one *can* prove Fubini's Theorem for functions like the $\tilde{f}$ we are using, but it does not provide that proof. I have not seen a proof of this case which does not involve measure theory (advanced real analysis), so I am not even going to tell you that you could look this one up. The rest of the proof isn't bad.

Examples 2, 3, 4, and 5 are the meat of the section. You should try to work through all of them. Notice that in each case, you have the option of whether to integrate with respect to x or y first. In examples 2 and 3, the book makes the choice it does because the region is simple in that direction. You could do either one in the opposite order, but you would need a sum of two integrals (why?). In example 4, the choice is forced because if you try to integrate with respect to y first, the integrand is unmanageable. In exercise 5, you could integrate in either order but this shows you how to switch the order for an interesting domain.

Examples 6 and 7 are ecology and geometry applications of double integrals. The integration in these examples isn't bad. The setup is the key point.

Example 8 is useful because it shows a way to bound an integral that you may not otherwise be able to compute. Example 9 uses integration to find the average value of a function (an extension of the same idea from Calc I).

Example 10 is not particularly interesting. If you have the idea of splitting up a domain already, this example will not add to your enlightenment.