Name: Key

Calculus 251:C3 Quiz #26 - 7/15/2020 Topic: Section 17.3

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

10 pts (1) Let $\vec{F} = \langle 3x - e^{\sin y}\cos z, -3y + e^{\cos x}\sin z, 2x^2 + 2y^2 \rangle$, and let S be the upper hemisphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$ oriented with upward-pointing normal.

Use the Divergence Theorem to calculate the flux of $\vec{F}$ through S.

$\vec{\nabla}\cdot\vec{F} = 3 + (-3) + 0 = 0$, so we are happy. But S is not a closed surface, so we do have some work to do.

Let D be the disk $\{(x,y,0) : x^2+y^2 \leq 4\}$ in the xy-plane, oriented with upward-pointing normal. Now $S - D$ is a closed surface with outward-pointing normal. Let W be the region enclosed by $S-D$.

$$0 = \iiint_W (\vec{\nabla}\cdot\vec{F})\,dV = \iint_S \vec{F}\cdot d\vec{S} - \iint_D \vec{F}\cdot d\vec{S} \implies \iint_S \vec{F}\cdot d\vec{S} = \iint_D \vec{F}\cdot d\vec{S}$$

Parametrize D: $G(r,\theta) = \langle r\cos\theta, r\sin\theta, 0\rangle$, $0 \leq r \leq 2$, $0 \leq \theta \leq 2\pi$

$\vec{T}_r = \langle \cos\theta, \sin\theta, 0\rangle$, $\vec{T}_\theta = \langle -r\sin\theta, r\cos\theta, 0\rangle$, $\vec{N} = \langle 0, 0, r\rangle$ upward ✓

$\vec{F}(G(r,\theta)) = \langle \text{blob}, \text{blob}, 2r^2\cos^2\theta + 2r^2\sin^2\theta\rangle = \langle \text{blob}, \text{blob}, 2r^2\rangle$

$\vec{F}\cdot\vec{N} = 2r^3$

So $\iint_S \vec{F}\cdot d\vec{S} = \iint_D \vec{F}\cdot d\vec{S} = \int_0^{2\pi}\int_0^2 2r^3\,dr\,d\theta = 2\pi\left(\frac{r^4}{2}\Big|_0^2\right) = 2\pi\cdot 8 = 16\pi$