Name: Key

Calculus 251:C3 **Quiz #25 - 7/14/2020** **Topic: Section 17.2**

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

10 pts

(1) Let $\vec{F} = \langle y^2 + z^2, x^2 + z^2, x^2 + y^2 \rangle$, and let C be the square in the xy-plane with vertices $(0,0,0), (0,-2,0), (2,-2,0)$, and $(2,0,0)$ oriented counterclockwise when viewed from above.

Use Stokes' Theorem to calculate $\oint_C \vec{F} \cdot d\vec{r}$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y^2+z^2 & x^2+z^2 & x^2+y^2 \end{vmatrix} = (2y-2z)\hat{i} - (2x-2z)\hat{j} + (2x-2y)\hat{k}$$

$$= \langle 2y-2z, 2z-2x, 2x-2y \rangle$$

Let S be the square $[0,2] \times [-2,0] \times \{0\}$ in the xy-plane, upward normal

so $G(x,y) = (x, y, 0)$, $x \in [0,2]$, $y \in [-2, 0]$

$\vec{T}_x = \langle 1,0,0 \rangle$, $\vec{T}_y = \langle 0,1,0 \rangle$, $\vec{N} = \langle 0,0,1 \rangle$

$\vec{F} \cdot \vec{N} = 2x - 2y$

$$\text{So } \oint_C \vec{F} \cdot d\vec{r} \underset{\text{by Stokes'}}{=} \int_0^2 \int_{-2}^0 (2x-2y)\,dy\,dx = \int_0^2 \left[2xy - y^2\right]_{y=-2}^{y=0} dx$$

$$= \int_0^2 (0 - (-4x-4))\,dx = \int_0^2 (4x+4)\,dx = 2x^2 + 4x \Big|_0^2 = 16$$