Name: Key

Calculus 251:C3 Quiz #24 - 7/13/2020 Topic: Section 17.1

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

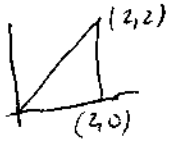
10 pts (1) Let $\vec{F} = \langle e^x - y, \sin x + y \rangle$, and let $\mathcal{D}$ be the region enclosed by the triangle with vertices $(0,0), (2,0)$, and $(2,2)$.

Verify Green's Theorem, i.e. show that $\oint_{\partial \mathcal{D}} \vec{F} \cdot d\vec{r} = \iint_{\mathcal{D}} \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dA$.

Hint: The right hand side is much easier. Do that one first.

RHS

$\frac{\partial F_2}{x} - \frac{\partial F_1}{y} = \cos x + 1$



$0 \le y \le x$

$0 \le x \le 2$

$\iint_D \left(\frac{\partial F_2}{x} - \frac{\partial F_1}{y} \right) dA = \int_0^2 \int_0^x (\cos x + 1)\, dy\, dx = \int_0^2 (y \cos x + y) \Big|_0^x dx = \int_0^2 (x \cos x + x)\, dx$

$= \left(x \sin x + \cos x + \frac{x^2}{2} \right) \Big|_0^2 = (2 \sin 2 + \cos 2 + 2) - (0 + 1 + 0)$

$= 2 \sin 2 + \cos 2 + 1$

$\int x \cos x\, dx = x \sin x + \cos x + C$

$u = x \quad v = \sin x$

$du = dx \quad dv = \cos x\, dx$

Bottom: $\vec{r}_1(t) = \langle t, 0 \rangle$, $t \in [0,2]$ $\quad \vec{F}(\vec{r}_1(t)) = \langle e^t, \sin t \rangle$

Right: $\vec{r}_2(t) = \langle 2, t \rangle$, $t \in [0,2]$ $\quad \vec{F}(\vec{r}_2(t)) = \langle e^2 - t, \sin 2 + t \rangle$

Top: $\vec{r}_3(t) = \langle 2-t, 2-t \rangle$, $t \in [0,2]$ $\quad \vec{F}(\vec{r}_3(t)) = \langle e^{2-t} - 2 + t, \sin(2-t) + 2 - t \rangle$

*Note: I was sneaky and made sure t went from 0 to 2 on all three sides.

***

$\oint_C \vec{F} \cdot d\vec{r} = \int_0^2 [e^t + \sin 2 + t - e^{2-t} + 2 - t - \sin(2-t) - 2 + t]\, dt$

$= \int_0^2 [e^t + \sin 2 + t - e^{2-t} - \sin(2-t)]\, dt$

$= \left[e^t + t \sin 2 + \frac{t^2}{2} + e^{2-t} - \cos(2-t) \right]_0^2$

$= (e^2 + 2 \sin 2 + 2 + 1 - 1) - (1 + 0 + 0 + e^2 - \cos 2)$

$= 2 \sin 2 + \cos 2 + 1$

*** left out two columns while I was copying from my scratch work

$\vec{r}_1'(t) = \langle 1, 0 \rangle \Rightarrow \vec{F}(\vec{r}_1(t)) \cdot \vec{r}_1'(t) = e^t$

$\vec{r}_2'(t) = \langle 0, 1 \rangle \Rightarrow \vec{F}(\vec{r}_2(t)) \cdot \vec{r}_2'(t) = \sin 2 + t$

$\vec{r}_3'(t) = \langle -1, -1 \rangle \Rightarrow \vec{F}(\vec{r}_3(t)) \cdot \vec{r}_3'(t) = -e^{2-t} + 2 - t - \sin(2-t) - 2 + t$