Name: Key

Calculus 251:C3 Quiz #23 - 7/7/2020 Topic: Section 16.5

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

10 pts (1) Let S be the part of the cylinder $x^2 + y^2 = 9$ with $2 \leq z \leq 8$ oriented with outward-pointing normal. Let $\vec{F} = \langle x, y, \ln(z^2+1) \rangle$.
Compute the flux of $\vec{F}$ through S

Parametrize in terms of z and θ (since r is constant)

$G(\theta, z) = (3\cos\theta, 3\sin\theta, z)$, $0 \leq \theta \leq 2\pi$, $2 \leq z \leq 8$

$\vec{T}_\theta = \langle -3\sin\theta, 3\cos\theta, 0 \rangle$

$\vec{T}_z = \langle 0, 0, 1 \rangle$

$\vec{F}(G(\theta, z)) = \langle 3\cos\theta, 3\sin\theta, \ln(z^2+1) \rangle$

$\vec{N} = \vec{T}_\theta \times \vec{T}_z = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3\sin\theta & 3\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = (3\cos\theta)\hat{i} + (3\sin\theta)\hat{j} + 0\hat{k}$ (points outward, so orientation is right)

$\vec{F} \cdot \vec{N} = 9\cos^2\theta + 9\sin^2\theta + 0 = 9$

$\iint_S \vec{F} \cdot d\vec{S} = \int_0^{2\pi} \int_2^8 9 \, dz \, d\theta = \int_0^{2\pi} 54 \, d\theta = 108\pi$