

Name: Key

Calculus 251:C3 Quiz #21 - 7/2/2020 Topic: Section 16.3

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

10 pts

(1) Let C be the part of an elliptical helix parametrized by

$$\vec{r}(t) = \left\langle 2 \cos t, 3 \sin t, \frac{2}{3\pi}t \right\rangle, 0 \leq t \leq \frac{9\pi}{2}.$$

Let $\vec{F} = \langle 3x^2 - 2y, z^2 - 2x, 2yz \rangle$ be a force field which acts on a particle traveling along the path C . Calculate the work done on the particle by \vec{F} .

Note: There are easy ways to do this problem and there are hard ways. Please don't do it the hard way.

Domain of \vec{F} is all of \mathbb{R}^3 - simply connected

$$\text{curl}(\vec{F}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x^2 - 2y & z^2 - 2x & 2yz \end{vmatrix} = (2z - 2z)\hat{i} - (0 - 0)\hat{j} + (-2 - (-2))\hat{k} = \vec{0} \Rightarrow \vec{F} \text{ conservative}$$

Finding a potential f :

$$f_x = 3x^2 - 2y$$

$$f = x^3 - 2xy + g(y, z)$$

$$f_y = z^2 - 2x$$

$$f_y = -2x + g_y$$

$$\text{so } g_y = z^2$$

$$g = yz^2 + h(z)$$

$$f = x^3 - 2xy + yz^2 + h(z)$$

$$f_z = 2yz$$

$$f_z = 2yz + h_z$$

$$\text{so } h_z = 0 \Rightarrow h(z) = C$$

$$f = x^3 - 2xy + yz^2 + C$$

So $f(x, y, z) = x^3 - 2xy + yz^2$ is a potential for \vec{F}

$$\vec{r}(0) = \langle 2, 0, 0 \rangle, \vec{r}\left(\frac{9\pi}{2}\right) = \langle 0, 3, 3 \rangle$$

$$\text{work} = \int_C \vec{F} \cdot d\vec{r} = f(0, 3, 3) - f(2, 0, 0) = 27 - 8 = 19$$