Name: Key

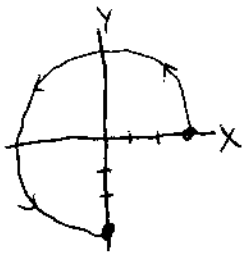
Calculus 251:C3 **Quiz #20 - 7/1/2020** **Topic: Section 16.2**

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

10 pts (1) Let C be the three quarters of the circle in the xy-plane with radius 3, centered at the origin, starting at $(3,0,0)$ and ending at $(0,-3,0)$. Let $\vec{F} = \langle xy, x^2, e^z \rangle$ be a force field which acts on a particle traveling along the path C.

Calculate the work done on the particle by $\vec{F}$.

Note that $\vec{F}$ is not conservative: $\frac{\partial F_2}{\partial x} = 2x \neq x = \frac{\partial F_1}{\partial y}$



C: $\vec{r}(t) = \langle 3\cos t, 3\sin t, 0 \rangle$, $0 \le t \le \frac{3\pi}{2}$

$\vec{F}(\vec{r}(t)) = \langle 9\sin t\cos t, 9\cos^2 t, 1 \rangle$

$\vec{r}\,'(t) = \langle -3\sin t, 3\cos t, 0 \rangle$

$\vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t) = \langle -27\sin^2 t\cos t, 27\cos^3 t, 0 \rangle$

So work $= \int_C \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t) = \int_0^{3\pi/2} (-27\sin^2 t\cos t + 27\cos^3 t)\, dt$

$= \int_0^{3\pi/2} (-27\sin^2 t\cos t + 27\cos t - 27\sin^2 t\cos t)\, dt$ * used $\cos^3 t = \cos t(1-\sin^2 t)$

$= \int_0^{3\pi/2} (-54\sin^2 t\cos t + 27\cos t)\, dt$

$= \left[-18\sin^3 t + 27\sin t \right]_0^{3\pi/2}$

$= \left[-18(-1)^3 + 27(-1) \right] - \left[-18(0) + 27(0) \right]$

$= -9$