

Name: Key

Calculus 251:C3 Quiz #19 - 6/30/2020 Topic: Section 16.1

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

10 pts

(1) Let $\vec{F} = \langle 2xe^z, z \cos(y), x^2e^x + \sin(y) \rangle$.

(a) Calculate $\text{div}(\vec{F})$.

(b) Calculate $\text{curl}(\vec{F})$.

(c) Is \vec{F} conservative? Why or why not?

(d) If your answer to (c) is yes, find a potential for \vec{F} .

$$\begin{aligned} \text{a) } \text{div}(\vec{F}) &= \frac{\partial}{\partial x}(2xe^z) + \frac{\partial}{\partial y}(z \cos y) + \frac{\partial}{\partial z}(x^2e^x + \sin y) \\ &= 2e^z - z \sin y \end{aligned}$$

$$\text{b) } \text{curl}(\vec{F}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xe^z & z \cos y & x^2e^x + \sin y \end{vmatrix}$$

$$= (\cos y - \cos y)\hat{i} - (2xe^x + x^2e^x - 2xe^z)\hat{j} + (0 - 0)\hat{k}$$

$$= (2xe^z - 2xe^x - x^2e^x)\hat{j}$$

c) No, because $\text{curl}(\vec{F}) \neq \vec{0}$

d) N/A

Confession: I meant to make the third component of \vec{F}

$x^2e^z + \sin y$. If I had not made this typo, \vec{F} would be conservative. If you want to practice, redo parts b, c, and d with this change.