

Name: Key

Calculus 251:C3 Quiz #18 - 6/29/2020 Topic: Section 15.6

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

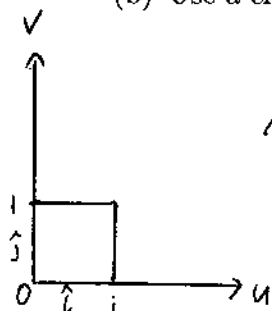
10 pts

(1) Let \mathcal{D} be the parallelogram in the xy -plane with vertices $(0,0)$, $(2,3)$, $(4,1)$, and $(6,4)$.

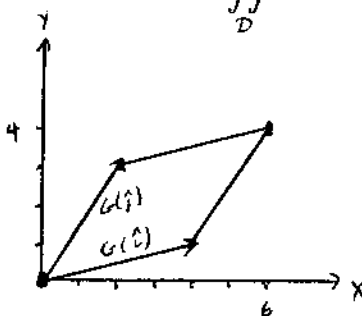
(a) Find a linear mapping G that maps $[0,1] \times [0,1]$ in the uv -plane onto \mathcal{D} .

(b) Use a change of variables to evaluate $\iint_{\mathcal{D}} e^{x+y} dA$.

a)



G



$$G(1,0) = (4,1)$$

$$G(0,1) = (2,3)$$

$$G(u,v) = (4u+2v, u+3v)$$

$$b) \text{ Jac}(G) = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} = \begin{vmatrix} 4 & 2 \\ 1 & 3 \end{vmatrix} = 12 - 2 = 10$$

$$\begin{aligned} \iint_{\mathcal{D}} e^{x+y} dx dy &= \int_0^1 \int_0^1 e^{(4u+2v)+(u+3v)} |\text{Jac}(G)| dv du \\ &= \int_0^1 \int_0^1 e^{5u+5v} (10) dv du \\ &= 10 \int_0^1 \int_0^1 e^{5u} e^{5v} dv du \\ &= 10 \int_0^1 e^{5u} du \int_0^1 e^{5v} dv \\ &= 10 \left(\frac{1}{5} e^{5u} \Big|_0^1 \right) \left(\frac{1}{5} e^{5v} \Big|_0^1 \right) \\ &= \frac{10}{25} (e^5 - 1)(e^5 - 1) \\ &= \frac{2}{5} (e^{10} - 2e^5 + 1) \\ &= \frac{2}{5} e^{10} - \frac{4}{5} e^5 + \frac{2}{5} \end{aligned}$$