Name: **Key**

Calculus 251:C3   Quiz #16 - 6/23/2020   Topic: Section 12.7

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

10 pts   (1) Consider the surface described by the equation $4x^2 + 4y^2 + z^2 = 16$ (in rectangular coordinates).

(a) Find an equation of the form $r = f(\theta, z)$ in cylindrical coordinates for this surface.

(b) Does this surface have axial symmetry? Justify your answer.

(c) Find an equation of the form $\rho = g(\theta, \phi)$ in spherical coordinates for this surface.

(d) If you were given a function $h(x, y, z)$ and asked to integrate the function over the region inside the surface, explain how would you decide which of the three coordinate systems to use. Note: you do not need to set up an integral. That's on tomorrow's quiz.

(a) $4(x^2 + y^2) + z^2 = 16$

$4r^2 + z^2 = 16$

$4r^2 = 16 - z^2$

$2r = \sqrt{16 - z^2}$

$r = \dfrac{\sqrt{16 - z^2}}{2}$

(b) Yes, because r is independent of θ

(c) $4\rho^2 \cos^2\theta \sin^2\varphi + 4\rho^2 \sin^2\theta \sin^2\varphi + \rho^2 \cos^2\varphi = 16$

$4\rho^2 \sin^2\varphi (\cos^2\theta + \sin^2\theta) + \rho^2 \cos^2\varphi = 16$

$\rho^2 (4\sin^2\varphi + \cos^2\varphi) = 16$

$\rho^2 = \dfrac{16}{4\sin^2\varphi + \cos^2\varphi}$

$\rho = \sqrt{\dfrac{16}{4\sin^2\varphi + \cos^2\varphi}}$

$\rho = \dfrac{4}{\sqrt{4\sin^2\varphi + \cos^2\varphi}}$

or

$\rho = \dfrac{4}{\sqrt{3\sin^2\varphi + 1}}$

d) The region has axial symmetry, so cylindrical coordinates are the natural choice. If the integrand is difficult to work with in cylindrical, that might persuade me to work in rectangular or spherical (whichever form is easiest to integrate).