

Name: liey

Calculus 251:C3 Quiz #13 - 6/17/2020 Topic: Section 15.1

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

10 pts

(1) Let \mathcal{R} be the rectangle $[1, 3] \times [1, 5]$ and let $f(x, y) = \frac{\ln(xy^2)}{x}$.

Find the volume below the surface $z = f(x, y)$ and above the region \mathcal{R} in the xy -plane.

$$\begin{aligned} & \int_1^3 \int_1^5 \frac{\ln(xy^2)}{x} dy dx \\ &= \int_1^3 \int_1^5 \frac{\ln x + 2 \ln y}{x} dy dx \\ &= \int_1^3 \int_1^5 \left(\frac{\ln x}{x} + \frac{2 \ln y}{x} \right) dy dx \\ &= \int_1^3 \int_1^5 \frac{\ln x}{x} dy dx + \int_1^3 \int_1^5 \frac{2 \ln y}{x} dy dx \\ &= \int_1^3 \frac{\ln x}{x} \left(\int_1^5 dy \right) dx + \int_1^3 \frac{2}{x} \left(\int_1^5 \ln y dy \right) dx \\ &= \left(\int_1^3 \frac{\ln x}{x} dx \right) \left(\int_1^5 dy \right) + \left(\int_1^3 \frac{2}{x} dx \right) \left(\int_1^5 \ln y dy \right) \\ &= \left(\frac{1}{2} \ln^2 x \Big|_1^3 \right) (y \Big|_1^5) + \left(2 \ln x \Big|_1^3 \right) \left((y \ln y - y) \Big|_1^5 \right) \\ &= \left(\frac{1}{2} \ln^2 3 \right) (4) + (2 \ln 3) \left((5 \ln 5 - 5) - (0 - 1) \right) \\ &= 2 \ln^2 3 + (2 \ln 3) (5 \ln 5 - 4) \\ &= 2 \ln 3 (\ln 3 + 5 \ln 5 - 4) \end{aligned}$$