

Name: Key

Calculus 251:C3 Quiz #12 - 6/16/2020 Topic: Polar coordinates

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

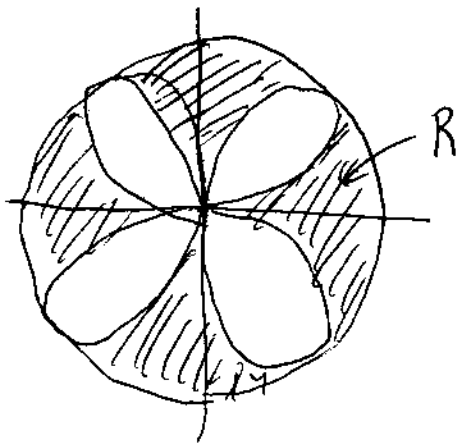
10 pts

(1) Let \mathcal{R} be the region inside the circle $r = 2$ and outside the "rose" $r = 2 \sin(2\theta)$.

(a) Sketch a graph of the two functions and shade the region \mathcal{R} .

(b) Calculate the area of \mathcal{R} .

Hint: $\int \sin^2(2x) dx = \frac{4x - \sin(4x)}{8} + C$



Will compute area in first quadrant,
then multiply by 4.

$$\begin{aligned} & 4 \cdot \frac{1}{2} \int_0^{\pi/2} [2^2 - (2 \sin(2\theta))^2] d\theta \\ &= 2 \cdot \int_0^{\pi/2} (4 - 4 \sin^2(2\theta)) d\theta \\ &= 8 \cdot \int_0^{\pi/2} (1 - \sin^2(2\theta)) d\theta \\ &= 8 \cdot \left[\theta - \frac{4\theta - \sin(4\theta)}{8} \right]_0^{\pi/2} \\ &= [8\theta - 4\theta + \sin(4\theta)]_0^{\pi/2} \\ &= [4\theta + \sin(4\theta)]_0^{\pi/2} \\ &= \left(4\left(\frac{\pi}{2}\right) + 0 \right) - (0 + 0) \\ &= 2\pi \end{aligned}$$