

Name: Key

Calculus 251:C3 Quiz #8 - 6/9/2020 Topic: Section 14.5

**Instructions.** Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

6 pts

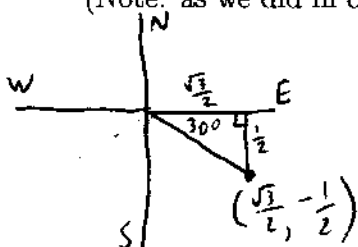
- (1) The temperature at the point  $(x, y, z)$  is given by  $T(x, y, z) = xy^3 - xz + y^2z^2 - 5$ .  
A particle travels along the helix parametrized by  $\vec{r}(t) = \langle 2 \sin t, 2 \cos t, t \rangle$ ,  $t \in \mathbb{R}$ .  
What is the rate of change along the particle's path at  $t = 0$ ?

$$\begin{aligned} \left. \frac{dT}{dt} \right|_{t=0} &= \nabla T_{\vec{r}(0)} \cdot \vec{r}'(0) \\ &= \langle 8, 0, 0 \rangle \cdot \langle 2, 0, 1 \rangle \\ &= \boxed{16} \end{aligned}$$

$$\begin{aligned} \nabla T &= \langle y^3 - z, 3xy^2 + 2yz^2, -x + 2y^2z \rangle \\ \vec{r}(0) &= \langle 0, 2, 0 \rangle \\ \nabla T_{\vec{r}(0)} &= \langle 8, 0, 0 \rangle \\ \vec{r}'(t) &= \langle 2 \cos t, -2 \sin t, 1 \rangle \\ \vec{r}'(0) &= \langle 2, 0, 1 \rangle \end{aligned}$$

4 pts

- (2) At the point  $P = (7, -2)$ , the function  $f(x, y)$  has gradient  $\nabla f(P) = \langle 2, 3 \rangle$ .  
What is the rate of change of  $f$  at point  $P$  in the direction  $30^\circ$  south of east?  
(Note: as we did in class, we will assume that the positive  $y$ -axis points north).



$\hat{u} = \langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \rangle$  is the unit direction vector

$$\begin{aligned} D_{\hat{u}} f(P) &= \nabla f(P) \cdot \hat{u} = \langle 2, 3 \rangle \cdot \langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \rangle \\ &= \sqrt{3} - \frac{3}{2} \end{aligned}$$