

Name: Key

Calculus 251:C3 Quiz #6 - 6/3/2020 Topic: Sections 14.1-14.2

**Instructions.** Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

- (1) For each function  $f$ , describe the contour curves. You should give a complete, concise, and clear English description. You may also graph several contour curves to support your answer, but the English description is required.

2 pts (a)  $f(x, y) = -3x + y$

The general level curve is  $-3x + y = c$  or  $y = 3x + c$ .  
The level curves are lines in  $\mathbb{R}^2$  with a slope of 3.

2 pts (b)  $f(x, y) = e^{x^2 - y - 2}$

The general level curve is  $e^{x^2 - y - 2} = c$ , or  $x^2 - y - 2 = \ln c$   
or  $y = x^2 - 2 - \ln c$

The level curves are upward opening parabolas with vertices on the  $y$ -axis. For larger values of  $c$ , the parabolas are lower on the graph and closer together.

2 pts (c)  $f(x, y) = \ln(x^2 + y^2)$

The general level curve is  $\ln(x^2 + y^2) = c$ , or  $x^2 + y^2 = e^c$

The level curves are circles centered at the origin. As  $c$  increases, the distance between level curves increases.

4 pts

(2) Calculate the following limit or show that it does not exist.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{2x+y}{\sqrt{x^2+y^2}}$$

along the x-axis:

$$\lim_{(x,y) \rightarrow (0,0)} \frac{2x+y}{\sqrt{x^2+y^2}} = \lim_{x \rightarrow 0} \frac{2x}{\sqrt{x^2}} = \lim_{x \rightarrow 0} \frac{2x}{|x|} \text{ which DNE } \left( \begin{array}{l} 2 \text{ from above,} \\ -2 \text{ from below} \end{array} \right)$$

along the y-axis:

$$\lim_{(x,y) \rightarrow (0,0)} \frac{2x+y}{\sqrt{x^2+y^2}} = \lim_{y \rightarrow 0} \frac{y}{\sqrt{y^2}} = \lim_{y \rightarrow 0} \frac{y}{|y|} \text{ which DNE } \left( \begin{array}{l} 1 \text{ from above} \\ -1 \text{ from below} \end{array} \right)$$

So just on the coordinate axes we have

four different limits, and we only need two

to show that  $\lim_{(x,y) \rightarrow (0,0)} \frac{2x+y}{\sqrt{x^2+y^2}}$  DNE.