

Name: Key

Calculus 251:C3 Quiz #2 - 5/27/2020 Topic: Sections 12.1-12.2

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

2 pts

(1) Given $\vec{v} = \langle \ln 10, \sec^2 7 \rangle$ and $\vec{w} = \langle \ln 5 - 4, \tan^2 7 \rangle$, compute $\vec{v} - \vec{w}$.

Component form of $\vec{v} - \vec{w}$: $\langle \ln 2 + 4, 1 \rangle$

$$\begin{aligned}\vec{v} - \vec{w} &= \langle \ln 10, \sec^2 7 \rangle + \langle -\ln 5 + 4, -\tan^2 7 \rangle \\ &= \langle \ln 10 - \ln 5 + 4, \sec^2 7 - \tan^2 7 \rangle \\ &= \langle \ln 2 + 4, 1 \rangle\end{aligned}$$

3 pts

(2) Find a parameterization of the line through $P(0, -4, 12)$ and $Q(-2, 4, 7)$

Parameterization: $\langle -2t, -4 + 8t, 12 - 5t \rangle$

$$\begin{aligned}\vec{v} &= \vec{PQ} = \langle -2, 8, -5 \rangle \\ \text{use } \vec{x}_0 &= \vec{OP} = \langle 0, -4, 12 \rangle \\ \vec{r}(t) &= \vec{x}_0 + t\vec{v} \\ &= \langle 0, -4, 12 \rangle + t\langle -2, 8, -5 \rangle \\ &= \langle -2t, -4 + 8t, 12 - 5t \rangle\end{aligned}$$

(3) Given the vectors $\vec{v} = 3\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{w} = 5\hat{i} - 2\hat{k}$

1 pt (a) Find $\|\vec{v}\|$

$$\begin{aligned}\|\vec{v}\| &= \sqrt{3^2 + (-1)^2 + 4^2} \\ &= \sqrt{9 + 1 + 16} \\ &= \sqrt{26}\end{aligned}$$

$$\|\vec{v}\| = \underline{\sqrt{26}}$$

1 pt (a) Find $\vec{v} - \vec{w}$

$$\begin{aligned}\vec{v} - \vec{w} &= (3\hat{i} - \hat{j} + 4\hat{k}) - (5\hat{i} - 2\hat{k}) \\ &= -2\hat{i} - \hat{j} + 6\hat{k}\end{aligned}$$

$$\vec{v} - \vec{w} = \underline{-2\hat{i} - \hat{j} + 6\hat{k}}$$

3 pts (4) Do the lines $\vec{r}_1(t) = \langle 2 - t, 1 + t, 6 \rangle$ and $\vec{r}_2(t) = \langle 3 + 2t, -2 - 4t, -3t \rangle$ intersect? If so, where?

Point of intersection (or "none"): none

need $s, t \in \mathbb{R}$ such that $\vec{r}_1(t) = \vec{r}_2(s)$

so ① $2 - t = 3 + 2s$

② $1 + t = -2 - 4s$

③ $6 = -3s$

③ $6 = -3s \Rightarrow s = -2$

now ② $1 + t = -2 - 4(-2)$

$1 + t = 6$

$t = 5$

substitute $s = -2, t = 5$ into ①

① $2 - 5 \stackrel{?}{=} 3 + 2(-2)$

$-3 \neq -1$

so the lines do not intersect