Math 251: Multivariable Calculus, Exam #3
Instructor: Blair Seidler

1. $\boxed{\text{20 pts}}$ Let $\mathcal{C}$ be the helix parametrized by $\vec{r}(t) = \langle 2\sin t, 2\cos t, \sqrt{5}\,t\rangle$ for $0 \le t \le 5\pi$.

   (a) Find the length of $\mathcal{C}$.

   (b) Calculate $\displaystyle\int_{\mathcal{C}} xyz\, ds$.

   (c) Let $\vec{F} = \langle 1, x^2, 0\rangle$. Calculate $\displaystyle\int_{\mathcal{C}} \vec{F}\cdot d\vec{r}$

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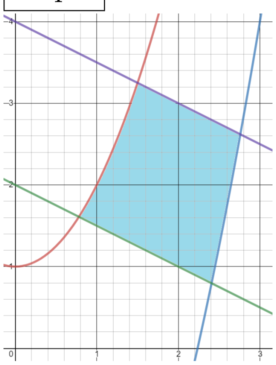
2. $\boxed{\text{20 pts}}$ Let $\mathcal{D}$ be the part of the first quadrant shaded in the diagram.



This region is bounded on the left and right by the curves $y - x^2 = 1$ and $y - x^2 = -5$, and on the top and bottom by the lines $x + 2y = 8$ and $x + 2y = 4$.

   (a) Find a rectangle $\mathcal{R}$ in the uv-plane and a map G such that $G(\mathcal{R}) = \mathcal{D}$. You may give either G or G^{-1}, but you must indicate which one your answer represents.

   (b) Calculate $\text{Jac}(G)$. You may give your answer in terms of x and y or in terms of u and v

   (c) Use a change of variables to calculate $\displaystyle\iint_{\mathcal{D}} (4x+1)e^{x^2+x+y}\, dx\, dy$.

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3. $\boxed{\text{18 pts}}$ Let $\vec{F} = \langle e^x \sin y, e^x \cos y - \cos(z^2), 2yz\sin(z^2)\rangle$.

   (a) Calculate $\text{div}(\vec{F})$.

   (b) Calculate $\text{curl}(\vec{F})$.

   (c) Is $\vec{F}$ conservative? Why or why not?

   (d) If your answer to (c) is yes, find a potential for $\vec{F}$.

4. $\boxed{\text{18 pts}}$ Let $\vec{F} = \langle 3x^2y, x^3 - 2yz, -y^2\rangle$.
Let $\mathcal{C}_1$ be the ellipse parametrized by $\vec{r}_1 = \langle 2\cos t, 5\sin t, 3\rangle$, $0 \le t \le 2\pi$.
Let $\mathcal{C}_2$ be the curve parametrized by $\vec{r}_2 = \left\langle 2\cos\left(\frac{\pi t}{4}\right), \frac{t^3}{25}, 2\ln(t+1)\right\rangle$, $0 \le t \le 5$.

   (a) Calculate $\displaystyle\int_{\mathcal{C}_1} \vec{F}\cdot d\vec{r}_1$.

   (b) Calculate $\displaystyle\int_{\mathcal{C}_2} \vec{F}\cdot d\vec{r}_2$.

5. $\boxed{\text{24 pts}}$ Let $\mathcal{S}$ be the surface $x^2 + y^2 = 16 - z$ for $z \ge 0$.

   (a) Parametrize the surface with a mapping $G(r,\theta)$.

   (b) Compute $\vec{T}_r, \vec{T}_\theta, \vec{N}$, orienting $\mathcal{S}$ with upward-pointing normal.

   (c) Find the surface area of $\mathcal{S}$.

   (d) Calculate the flux of $\vec{F} = \langle 0, 0, 3z\rangle$ across $\mathcal{S}$.