Math 251: Multivariable Calculus, Exam #2
Instructor: Blair Seidler

1. [12 pts] Find the point or points on the surface $z^2 = xy + 4$ closest to the origin.

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2. [16 pts] Calculate $\displaystyle\int_0^3 \int_0^{9-y^2} \frac{ye^{2x}}{9-x}\, dx\, dy$

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3. [10 pts] Let x, y, and z be related implicitly by the equation $xy^2 - 2xz + 5z^2 = 11$.
Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$

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4. [12 pts] Let $f(x, y) = x^3 + y^3 - 3xy + 15$. Find all critical points and critical values of f. Classify each critical point as a maximum, minimum, or saddle point.

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5. [16 pts] Let $\mathcal{R}$ be the rectangle $\{(x, y) : -2 \le x \le 2, 0 \le y \le 2\}$ and let $f(x, y) = \dfrac{y}{4 + x^2}$.
Calculate $\displaystyle\iint_{\mathcal{R}} f(x, y)\, dA$.

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6. [16 pts] The temperature in a room is modeled by $T(x, y, z) = xz^2 - 4y$. An insect flies through the room along the path $\vec{r}(t) = \left\langle e^{t-1}, \frac{4}{\pi} \sin\left(\frac{\pi}{2}t\right), 4 - t \right\rangle$.

   (a) Calculate $\nabla T(x, y, z)$.

   (b) Find a parametrization of the line tangent to the insect's path at $t = 2$.

   (c) What is the rate of change in temperature for the insect at $t = 2$? (i.e. what is the change in temperature along the insect's path at that moment?)

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7. [18 pts] Let $\mathcal{S}$ be the sphere of radius 2 centered at the origin.
Let $\mathcal{P}$ be the paraboloid $x^2 + y^2 = 3z$.
Let $\mathcal{W}$ be the region inside $\mathcal{S}$ and above $\mathcal{P}$ (so $\mathcal{W}$ includes part of the positive z-axis).

   (a) Write $\iiint_{\mathcal{W}} z \, dV$ as an iterated integral in rectangular coordinates.

   (b) Write $\iiint_{\mathcal{W}} z \, dV$ as an iterated integral in cylindrical coordinates.

   (c) Write $\iiint_{\mathcal{W}} z \, dV$ as an iterated integral in spherical coordinates (Warning: this one is significanty more difficult than the first two).

   (d) Choose any one of these integrals and use it to calculate $\iiint_{\mathcal{W}} z \, dV$.
