Math 251: Multivariable Calculus, Exam #1
Instructor: Blair Seidler

1. The planes $\mathcal{P}_1$ and $\mathcal{P}_2$ are described by the following equations.
   $\mathcal{P}_1$: $x - 2y + 4z = 2$
   $\mathcal{P}_2$: $x + y - 2z = 5$

[6 pts] (a) Find the angle between $\mathcal{P}_1$ and $\mathcal{P}_2$.

[9 pts] (b) The planes $\mathcal{P}_1$ and $\mathcal{P}_2$ intersect in the line $\mathcal{L}$. Find a parametrization of $\mathcal{L}$

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2. A particle travels on a path which satisfies the equation $\dfrac{d\vec{r}}{dt} = \left\langle e^{t-2}, 3\pi\cos\left(\dfrac{\pi}{4}t\right), t^2 \right\rangle$ for all $t \geq 0$.

[8 pts] (a) Find the general solution $\vec{r}(t)$ of the equation above which gives the position of the particle.

[5 pts] (b) Find the particular solution $\vec{r}(t)$ when $\vec{r}(2) = \langle 4, 10, 3\rangle$.

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3. Calculate each limit or show that the limit does not exist.

[6 pts] (a) $\displaystyle\lim_{(x,y)\to(2,0)} \frac{x^2\sin(3y)}{y}$

[6 pts] (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + xy + y^2}$

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4. Let $\mathcal{L}_1$ and $\mathcal{L}_2$ be two lines in $\mathbb{R}^3$ representing the position of two particles at time t with the following parametrizations:
   $\mathcal{L}_1$: $\vec{r}_1(t) = \langle 2 + 3t, -4 + \lambda t, -4\rangle$
   $\mathcal{L}_2$: $\vec{r}_2(t) = \langle 18 - t, 4 - 4t, -12 + 2t\rangle$

[3 pts] (a) For what value of λ do the lines intersect?

[3 pts] (b) What is the point of intersection?

[2 pts] (c) Do the particles collide?

[7 pts] (d) Find an equation of the plane containing both lines.

5. Consider the function $f(x, y) = \ln(x - y^2 + 1)$

5 pts (a) Sketch any 3 level curves of the function. Label each curve with the appropriate function value.

3 pts (b) Give a complete and concise English description of the set of all level curves of $f(x, y)$.

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6. Let $\vec{v} = \langle 2, -4, 8 \rangle$ and $\vec{w} = \langle 1, a, b \rangle$.

5 pts (a) For what values of a and b are $\vec{v}$ and $\vec{w}$ parallel?

8 pts (b) For what values of a and b are $\vec{v}$ and $\vec{w}$ perpendicular?

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7. Let $\vec{r}(t) = (3\cos t)\hat{\mathbf{i}} + (3\sin t)\hat{\mathbf{j}} + \sqrt{7}t\hat{\mathbf{k}}$.

8 pts (a) Find the tangent vector to $\vec{r}(t)$ at $t = 0$.

6 pts (b) Find the arc length of $\vec{r}(t)$ from $t = 0$ to $t = \pi$.

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8. Let $\beta = \dfrac{1 + \sqrt[3]{8.03}}{\sqrt{15.99}}$

10 pts Use an appropriate function $f(x, y)$ and linear approximation to estimate the value of β. Your answer should be a single fraction in lowest terms.
