READING COURSE

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Algebraic geometry is the study of geometric figures defined by polynomial equations, also called algebraic varieties. Much classical algebraic geometry was motivated by enumerative geometry, which aims to predict how many geometric figures of a specified type satisfy a given list of conditions. For example, how many curves of a specified degree contain a given list of points in the plane?

A modern strategy is to construct an algebraic variety called a moduli space that has one point for each of the figures of the given type, and interpret the list of conditions as a set of polynomial equations on the moduli space. The enumerative geometric problem then becomes equivalent to deciding the number of common solutions to a list of polynomial equations, or to counting the number of points in the intersection of a list of subvarieties of the moduli space.

The main tool for counting the number of intersection points of subvarieties in an algebraic variety X is the (Chow) cohomology ring of X. Each subvariety of X defines a class in this ring, and the class of the intersection of two (transversal) subvarieties is equal to the product of their individual classes. For example, if two subvarieties intersect transversally in 7 points, then the product of their classes is equal to 7 times the class of a single point.

A successful solution to an enumerative geometric problem may involve several steps, including: (1) construct a well behaved moduli space, (2) understand the cohomology ring of this moduli space, e.g. in terms of generators and relations, (3) calculate the classes of the relevant subvarieties representing the list of conditions, (4) multiply these classes and extract the number of solutions. Ideally one would like to understand the moduli space and its cohomology so well that the whole process can be expressed in combinatorial terms. This gives rise to rich interplay between enumerative geometry and combinatorics.

One of the simplest examples of a moduli space is the Grassmann variety of subspaces in a complex vector space. Here the rich theory of Schubert calculus is available to account for geometric as well as combinatorial aspects of all of the above steps. The Schubert calculus of Grassmannians can be studied without much background, which makes it an excellent subject for a reading course. While this is my first suggestion, I would also consider reading courses in other topics within algebraic geometry and the parts of combinatorics that I am interested in.

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