Practice problems for time-dependent BVP's

Posted by request from several students.

1: Find the function $u(x,t)$ defined for $0 \le x \le 2$ and $t \ge 0$ satisfying:

$3u_{xx} - \frac{x}{2}\sin(t) = u_t$, $0 < x < 2$, $t > 0$;

$u(0,t) = 0$, $u(2,t) = \cos(t)$, $t > 0$;

$u(x,0) = \frac{x}{2} + 3\sin(\pi x)$, $0 < x < 2$.

2: Find the function $u(x,t)$ defined for $0 \le x \le \pi$ and $t \ge 0$ satisfying:

$u_{xx} = u_t$, $0 < x < \pi$, $t > 0$;

$u(0,t) = 0$, $u(\pi,t) = \sin(t)$, $t > 0$;

$u(x,0) = 0$, $0 < x < \pi$.

Solutions:

1. $u(x,t) = \frac{x}{2}\cos(t) + 3\sin(\pi x)\exp(-3\pi^2 t)$.

2. $u(x,t) = \frac{x}{\pi}\sin(t) + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n(n^4+1)}\left[\sin(t) + n^2\cos(t) - n^2\exp(-n^2 t)\right]\sin(nx)$.