Calculus 421, Spring 2015, Final Exam Formula Sheet

The Laplacian in polar coordinates:

$$\nabla^2 u \;=\; u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} \;=\; \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$$

Euler's equation and general solution:

$$x^2 y'' + axy' + by = 0$$

$$y(x) = c_1 x^{\beta_1} + c_2 x^{\beta_2} \qquad \text{or} \qquad y(x) = c_1 x^{\beta} + c_2 x^{\beta}\ln(x)$$

Fourier integral:

$$f(x) = \frac{1}{\pi}\int_0^\infty \big[A(\alpha)\cos(\alpha x) + B(\alpha)\sin(\alpha x)\big]d\alpha$$

$$A(\alpha) = \int_{-\infty}^{\infty} f(x)\cos(\alpha x)dx \quad ; \quad B(\alpha) = \int_{-\infty}^{\infty} f(x)\sin(\alpha x)dx$$

Fourier transform:

$$\mathcal{F}\{f(x)\} = \int_{-\infty}^{\infty} f(x)e^{i\alpha x}dx$$

$$\mathcal{F}^{-1}\{F(\alpha)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\alpha)e^{-i\alpha x}d\alpha$$

$$\mathcal{F}\{f'(x)\} = -i\alpha\,\mathcal{F}\{f(x)\}$$

Fourier cosine transform:

$$\mathcal{F}_c\{f(x)\} = \int_0^\infty f(x)\cos(\alpha x)dx$$

$$\mathcal{F}_c^{-1}\{F(\alpha)\} = \frac{2}{\pi}\int_0^\infty F(\alpha)\cos(\alpha x)d\alpha$$

$$\mathcal{F}_c\{f''(x)\} = -\alpha^2\,\mathcal{F}_c\{f(x)\} - f'(0)$$

Fourier sine transform:

$$\mathcal{F}_s\{f(x)\} = \int_0^\infty f(x)\sin(\alpha x)dx$$

$$\mathcal{F}_s^{-1}\{F(\alpha)\} = \frac{2}{\pi}\int_0^\infty F(\alpha)\sin(\alpha x)d\alpha$$

$$\mathcal{F}_s\{f''(x)\} = -\alpha^2\,\mathcal{F}_s\{f(x)\} + \alpha f(0)$$