

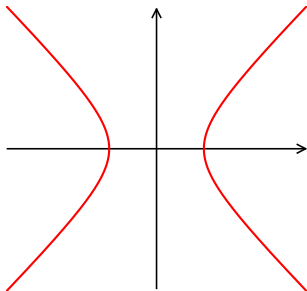
Algebraic Geometry

Affine algebraic variety a.k.a. **Zariski-closed subset of $\mathbb{C}^n$:**

$$Z(f_1, \dots, f_m) = \{x \in \mathbb{C}^n \mid f_1(x) = \dots = f_m(x) = 0\}$$

where $f_1, \dots, f_m \in \mathbb{C}[x_1, \dots, x_n]$ are polynomials in $x = (x_1, \dots, x_n)$.

$$Z(x^2 - y^2 - 1) \subset \mathbb{C}^2 :$$

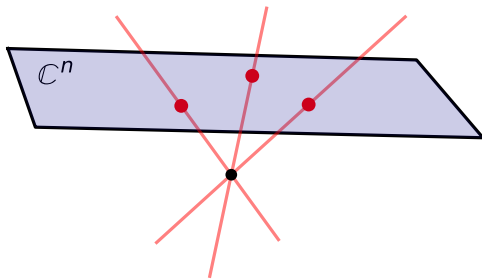


Affine algebraic varieties are **not compact**.

Projective space

$$\mathbb{P}^n = \mathbb{C}^n \cup \{\text{points at } \infty\} = \{\text{lines through } 0 \text{ in } \mathbb{C}^{n+1}\}$$

Coordinate neighborhood $\mathbb{C}^n \subset \mathbb{P}^n$:

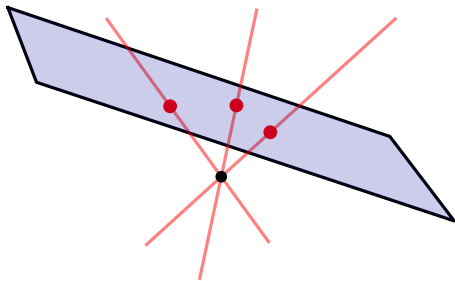


$$\text{point in } \mathbb{P}^n \longleftrightarrow \text{line in } \mathbb{C}^{n+1}$$

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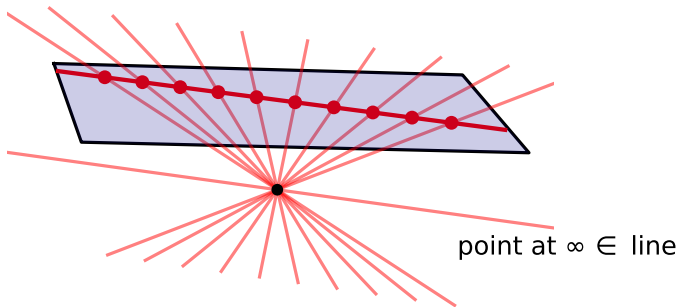


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Projective space

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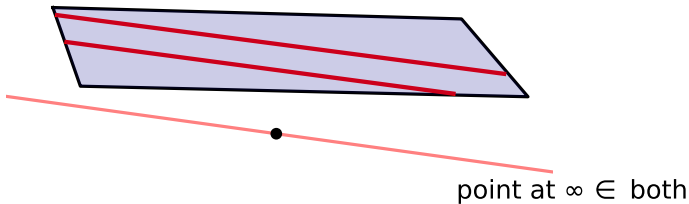


$$\text{line in } \mathbb{P}^n \longleftrightarrow \text{plane in } \mathbb{C}^{n+1}$$

Projective space

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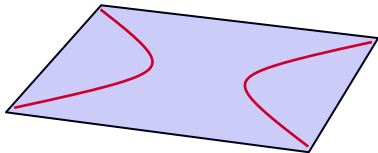
Coordinate neighborhood $\mathbb{C}^n \subset \mathbb{P}^n$:



two parallel lines in $\mathbb{P}^n$ meet at ∞

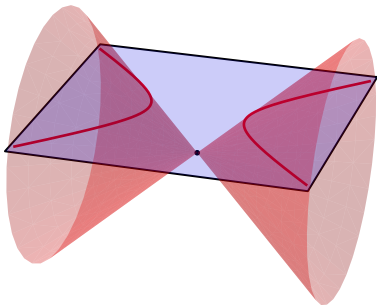
Projective variety = Zariski-closed subset of $\mathbb{P}^n$

Projective closure of hyperbola in $\mathbb{P}^2$:



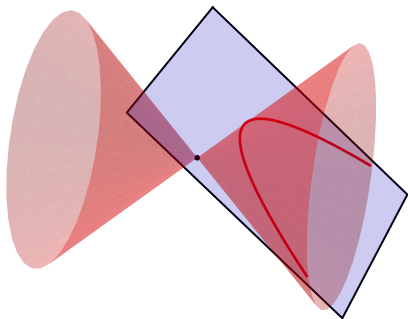
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