ALGEBRAIC GEOMETRY PROBLEMS

Let k be an algebraically closed field and let $\mathbb{A}^n = k^n$ be affine n-space over k.

Problem 1. Define the ideal $I = (y^2 + 2xy^2 + x^2 - x^4, x^2 - x^3) \subset k[x, y]$. Find (generators of) the radical $\sqrt{I}$.

Problem 2. Show that $W = \{(x, y, z) \in \mathbb{A}^3 \mid x^2 = y^3 \text{ and } y^2 = z^3\}$ is a Zariski closed subset of $\mathbb{A}^3$, find $I(W)$, and show that $I(W)$ is a prime ideal in $k[x, y, z]$. *Hint: Construct a ring homomorphism $k[x, y, z] \to k[T]$ with kernel $I(W)$.*

Problem 3. [Hartshorne I.1.2 and I.1.11]
(a) Show that $X = \{(t, t^2, t^3) \in \mathbb{A}^3 \mid t \in k\}$ is Zariski closed in $\mathbb{A}^3$ and find $I(X)$.
(b) Same for $Y = \{(t^3, t^4, t^5) \in \mathbb{A}^3 \mid t \in k\}$.
(c) Show that $I(Y)$ can't be generated by less than three polynomials.
Hint: Is $I(Y)$ a graded ideal? Are you sure??