Problem 1.
Let $G$ be a group of order 10000. Prove that $G$ is not simple. (Hint: Prove first that $G$ is not isomorphic to a subgroup of $S_{16}$.)

Problem 2.
Prove that the function $\phi(a + ib) = a^2 + b^2$ makes the ring $\mathbb{Z}[i] = \mathbb{Z}[\sqrt{-1}]$ of Gaussian integers into an Euclidean domain.

Problem 3.
Let $A$ be a real symmetric matrix. Prove that all eigenvalues of $A$ are non-negative if and only if there exists a real matrix $P$ such that $A = P^T P$.

Problem 4.
Find the cardinality of any maximal set of non-similar nilpotent matrices in $M_7(\mathbb{C})$.

Problem 5.
Let $\phi : \mathbb{Z}^3 \to \mathbb{Z}^4$ be the homomorphism of abelian groups defined by
\[
\begin{align*}
\phi(e_1) &= 6e_1 + 3e_2 + 3e_3 + 3e_4 \\
\phi(e_2) &= 4e_1 + 3e_2 + e_3 + e_4 \\
\phi(e_3) &= 2e_1 + 3e_2 + 17e_3 + 8e_4
\end{align*}
\]
Find a product of cyclic groups that is isomorphic to the cokernel $\mathbb{Z}^4/\phi(\mathbb{Z}^3)$.

Problem 6.
Let $A = \mathbb{C}[S_3]$ be the group ring of the symmetric group $S_3$ over the field of complex numbers. Find a direct product of matrix rings over division rings that is isomorphic to $A$.

Problem 7.
Let $p$ be a prime number. Classify, up to isomorphism, all groups of order $2p$. 