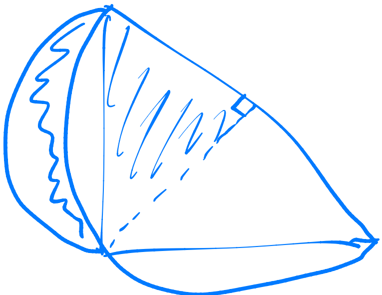
Last time: Hippocrates's

Quadrature of the Lune



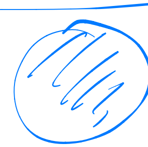
Big Three Construction Problems from Antiquity

(I) Square the Circle.

(II) Doubling a Cube

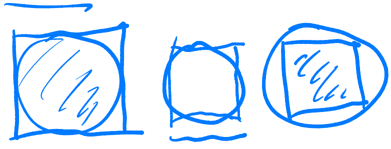
(III) Trisecting an angle.

(I) Given Circle



Can we construct a square with exactly the same area?

Clearly such a square exists:



Area (circle) = $\pi r^2 = \pi$. $\sqrt{\pi}$.

$r = 1$.

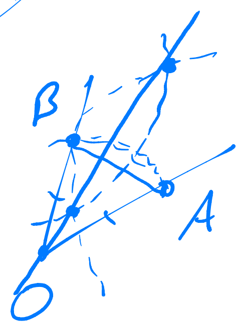
So: to construct a square with Exactly the same area amounts to constructing a Length of:

$$\sqrt{\pi} \times \sqrt{\pi}$$

The main question becomes: can you construct a length of sqrt Pi?

III Trisecting an angle?

Bisecting an angle.



① Draw ⊙ c O r $\overline{OA}$ ⇒ B

Notice: ΔAOB is osceles. Want: mid pt A to B.

② Draw ⊙ c A r AB

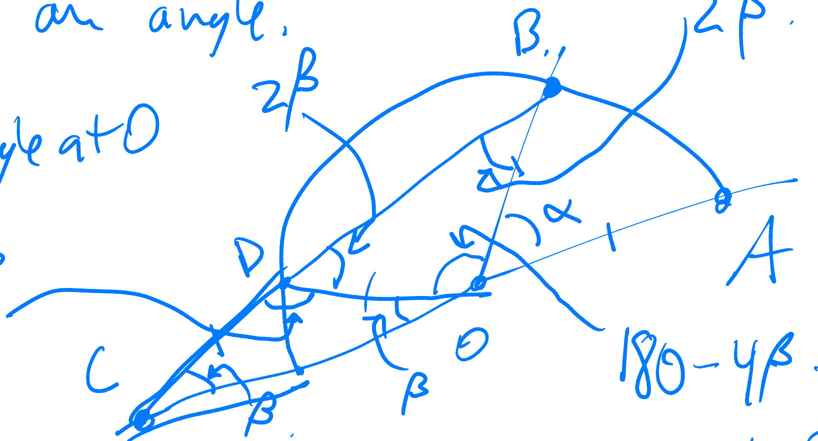
③ Draw ⊙ c B r AB,

④ Draw line through those two pts of intersection.

Then that line IS the angle bisector.

Claim: Here is how to trisect an angle.

Given angle at O



① Draw $\bigodot_{O, r = \overline{OA}}$, intersect at B, $\overline{OA} = \overline{OB}$.

② ~~Mark length $\overline{OB}$ on your~~ ruler,

and hold your ruler so that the end is on the line OA (at the point C), marked length stays on the circle (at D), and the ruler passes through B.

Then C, D, and B are collinear. And $\overline{CD} = \overline{OB} = \overline{OA} = \overline{OD}$.

Let $\angle OCD = \beta$.

$\angle ODC = 180 - 2\beta$.

$\Rightarrow \angle BDO = 2\beta$.

$$\Rightarrow \measuredangle OBD = 2\beta$$

$$\Rightarrow \measuredangle BOD = 180 - 4\beta.$$

$$\alpha + (\cancel{180} - 4\beta) + \beta = \cancel{180}.$$

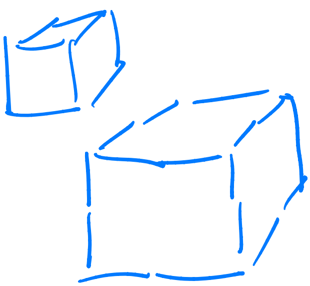
$$0$$

$$\alpha = 3\beta \Rightarrow \boxed{\beta = \frac{\alpha}{3}}$$

(II) Double A Cube?

1x1x1 cube,

double side:

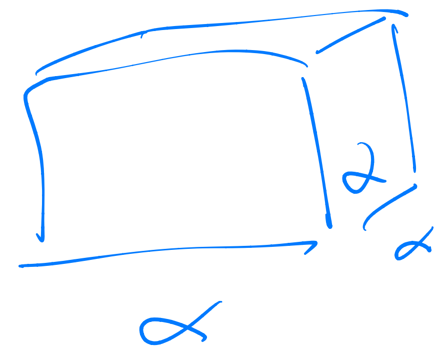


2x2x2 cube,

Volume = 8

Original volume was multiplied by $8 = 2^3$ cubed,

Goal: Double volume.



$$\alpha \cdot \alpha \cdot \alpha = 2$$

$$\Rightarrow \alpha = \sqrt[3]{2}.$$

Aside: $\sqrt[3]{2} = \left(2^{1/12}\right)^4$

$= 2^{1/3}$

Musical interval: C → E (major third).

$$\underbrace{C \to E \to G\# \to C}_{\text{"}Ab\text{"}}.$$

(augmented chord).