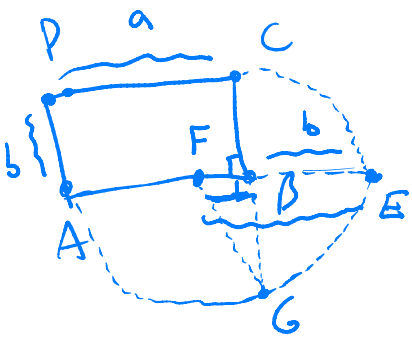
Recall: Quadrature = Squaring.

= making a square with same area.

(Constructing = with compass + straight edge).

Thm: Rectangle is Quadrable.



Pf: ⓪ start with a rectangle ABCD

① Draw $\odot(B, \overline{BC})$

② Extend AB to E ($\overline{AE} = a+b$)

③ Bisect $\overline{AE}$ at F

$\overline{AF} = \overline{FE} = \frac{a+b}{2}$.

④ Draw $\odot(F, \overline{AF})$

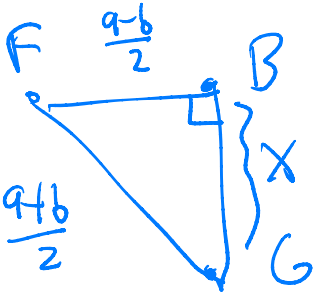
$\overline{FG} = \overline{AF} = \frac{a+b}{2}$.

⑤ Extend BC to G.

⑥ Finish making a square

Calculate: $\overline{BF} = \frac{a+b}{2} - b$

$= \frac{a-b}{2}$.



Pyth Thm $\Rightarrow$

$$X^2 + \left(\frac{a-b}{2}\right)^2 = \left(\frac{a+b}{2}\right)^2.$$

Solve for X. $= ?$.

$\Rightarrow$ $\left(\frac{a+b}{2}\right)\left(\frac{a+b}{2}\right) = \frac{a^2+2ab+b^2}{4}$

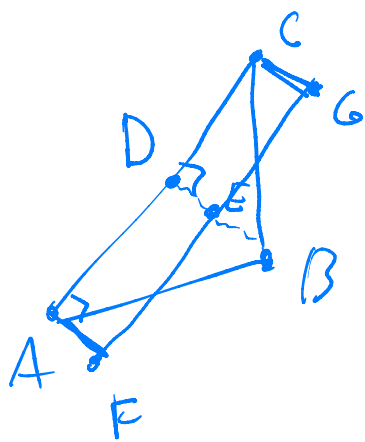
$$X^2 + \frac{a^2-2ab+b^2}{4} = \frac{a^2+2ab+b^2}{4}.$$

$$4X^2 + a^2 - 2ab + b^2 = a^2 + 2ab + b^2.$$
$$+2ab \qquad\qquad +2ab$$

$$4X^2 = 4ab.$$

$X = \sqrt{ab}$. $\qquad X^2 = ab$. QED.

Thm: The triangle is quadrable.



pf: ① Given $\triangle ABC$

② Draw $\perp$ to AC thru B. at D.

think about AC as the "base" and BD as "height"

③ Bisect BD at E.

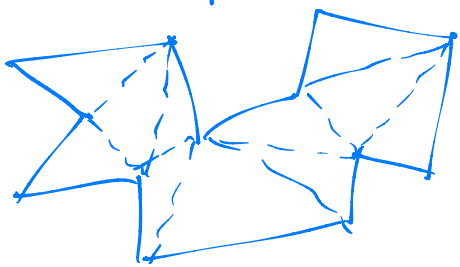
④ Draw rect with base AC, height DE

Claim: $\text{Area}(AFGC) = \text{Area}(\triangle ADC)$, $= \frac{1}{2} \text{base} \cdot \text{height}$

& We're DONE.

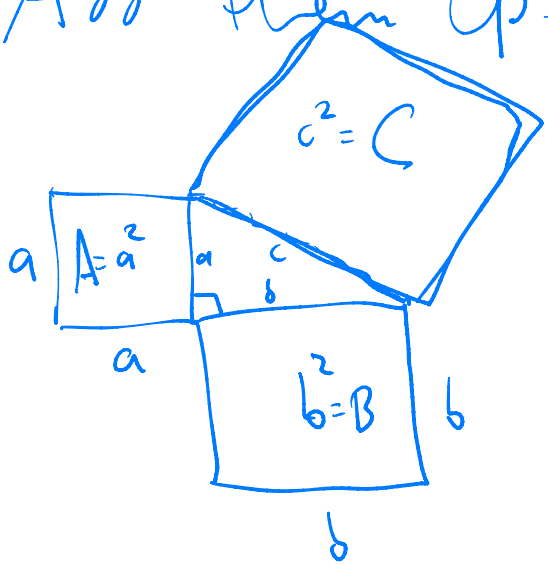
We have reduced the quadrature of the Triangle to that of the Rectangle. The latter we know to be quadrable!

Thm: Every Rectilinear Shape (polygon).
Is Quadrable.



Pf: ① Triangulate
(break polygon into triangles).
Square each ~~Δ~~ triangle.

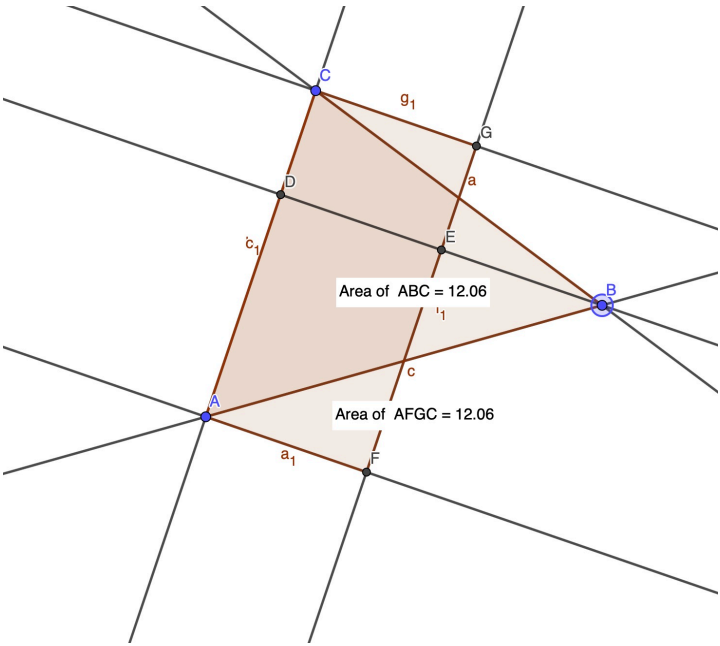
② Add them up. How???



$a^2 + b^2 = c^2$.

$A + B = C$.

QED.


C
g1
G
a
D
c1
E
Area of ABC = 12.06
B
t1
c
A
Area of AFGC = 12.06
a1
F