Pythagorean geometry, music theory, number theory

Last time:

$1 = 1$

$1+3=4$

$1+3+5=9$

$1+3+5+7=16$

$1+3+5+7+9=25$

Thm: $\forall n \in \mathbb{N}$, $\underbrace{1+3+5+\cdots+(2n-1)}_{n \text{ odd numbers}} = n^2$.

"in"

Principle of Mathematical Induction.

Want to prove some fact, $P : \mathbb{N} \to \text{Bool}$ (TRUE/FALSE)

① $P(1)$ is TRUE ② $\forall k \in \mathbb{N}$, if $\overset{n}{P(k)} \to \overset{n}{P(k+1)}$.

Then: $\forall n$, $P(n)$ is TRUE.

In Thm, $P(n) =$ whether $1+3+\cdots+(2n-1) = n^2$?

Pf: ① $P(1) =$ TRUE?. $n=1$, $1 = 1^2$ ✓.

② Given k, Assume $P(k)$ is TRUE, $\underline{1+3+\cdots(2k-1) = k^2}$.

Need to show: $(n=k+1)$. $P(k+1)$:

$$\underbrace{1+3+\cdots+(2k-1)}_{=\,k^2}+\big(2(k+1)-1\big) \overset{?}{=} \underbrace{(k+1)^2}_{=\,k^2+2k+1}.$$

$$k^2+2k+1 = k^2 + \big(2(k+1)-1\big) \quad \checkmark$$

By the principle of mathematical induction, P(n) holds for all n

QED.

Thm: All cows have exactly the same number of spots.

$P(n)$: Any collection of n cows has exactly the same # of spots.

Pf: (1) When $n=1$:

$P(1)$. One cow has the same number of spots as itself. ✓

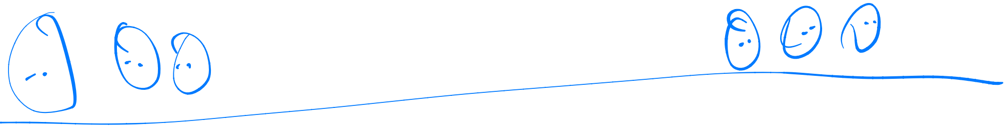
(2) Assume $P(k)$: Any collection of k cows has exactly the same number of spots.

What about $n=k+1$?

logic is good for $k \geq 2$ NOT $k=1$.

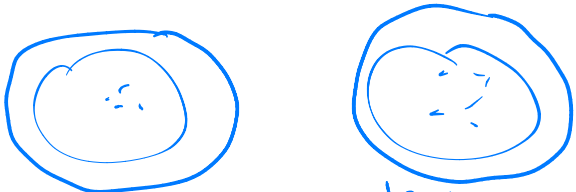
The first k cows all have the same number of spots. So do the last k cows. And hence all k+1 cows have the same number of spots. QED

P(3)

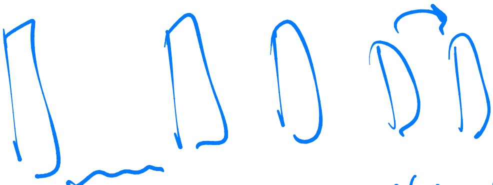


P(2) Not true.

The induction fails to go from n = 1 to n = 1 +1 =2. There's no overlap in the case n=2 in the middle cows.



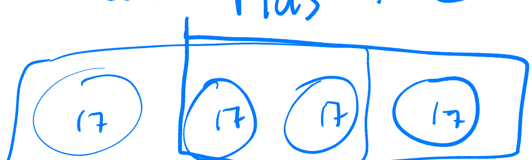
k=1 k+1=2



P(1) P(2) P(3) P(4) P(5)

Proved: if the 10th domino falls, then so must the 11th. Similarly, if the 2nd domino falls, then so must the 3rd, and forever.
We failed to prove that the first domino falling knocks over the 2nd.

For n=3, Assuming <u>any</u> collection of 3 cows has the same number of spots.



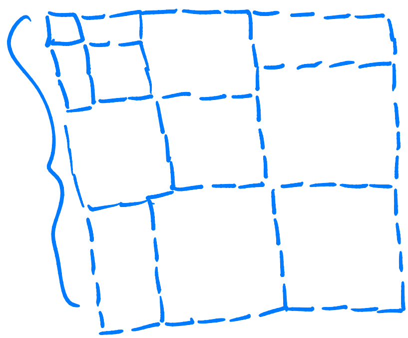
all have same # of spots (=17).

We assume that ANY collection of 3 cows has the same number of spots.
Look at 4 cows. The first three
t

Let $\mathcal{C} = \{\text{all cows}\}$.

$P : \mathbb{N} \to \text{Bool} : n \mapsto \forall P \subseteq \mathcal{C}, |P| = n$
$\Rightarrow$ all

$1 \cdot 1^2 + 2 \cdot 2^2 + 3 \cdot 3^2 + 4 \cdot 4^2$



$$1^3 = 1$$

$$1^3 + 2^3 = 9$$

$$1^3 + 2^3 + 3^3 = 36$$

$$1^3 + 2^3 + 3^3 + 4^3 = \underline{100}$$
