1400 | 1530s | 1540s | 1630s | 1660s Newton. Fluxions | Euler (1707–1783) | Gauss 1800

Mathematicians 1500

del Ferro → Fior → Tartaglia → Cardano → Ferrari

1600 Descartes — Fermat — Leibniz Calculus 1684 — 1700 — 1711 — St Peter. (Russia)

1687 Principia

Bernoulli — Johann, Jakob. → Marquis de l'Hopital.

Archimedes.

$y=x^2$

$y=x^3$

$$\frac{f(x+h)-f(x)}{(x+h)-x} = \frac{0}{0}.$$

Series: adding stuff up (Zeno).

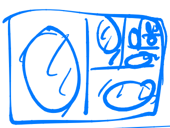
"harmonic" Series: $\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\cdots = ?$

$= \infty$

$\geq$

pf: $\frac{1}{1}+\frac{1}{2}+\underbrace{\frac{1}{4}+\frac{1}{4}}_{\frac{1}{2}}+\underbrace{\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}}_{\frac{1}{2}}+\underbrace{\frac{1}{16}+\cdots\frac{1}{16}}_{8;\ \frac{1}{2}}+\cdots$

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots = 1$



Leibniz $\sum \frac{1}{\Delta}$ Pythagoreans: 1, 3, 6

$$\frac{1}{1}+\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}+\frac{1}{21}+\cdots$$

$$\boxed{1+2+3+\cdots+n = \frac{n(n+1)}{2}.}$$

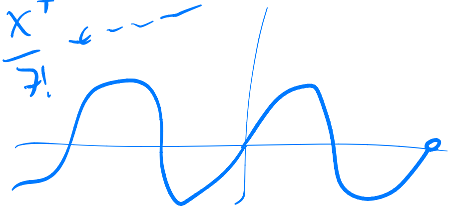
$$= 2.$$

Bernoulli's: $\sum \frac{1}{\square} = \frac{1}{1}+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\cdots \rightarrow ?? \approx 1.64\ldots$

"Basel Problem"

(Taylor 1715).

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$



$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} \cdots$$

$$\boxed{\begin{aligned} i^2 &= -1 \\ i^3 &= -i \\ i^4 &= +1. \end{aligned}}$$

Euler

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$$

$x = i\theta$

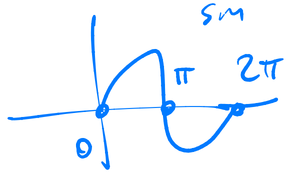
$$e^{i\theta} = 1 + i\theta - \frac{\theta^2}{2!} - \frac{i\theta^3}{3!} + \frac{\theta^4}{4!} + \frac{i\theta^5}{5!} - \cdots$$

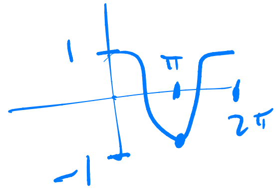
$$= \cos\theta + i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} \cdots\right).$$

$$e^{i\theta} = \cos\theta + i\sin\theta.$$

$$\boxed{\theta = \pi?}$$

$$e^{i\pi} = \underbrace{\cos\pi}_{-1} + i\,\cancel{\sin\pi}^{\,0}$$





$\sqrt{-1}$ ↓ circle ↙

$$e^{i\pi} + 1 = 0$$

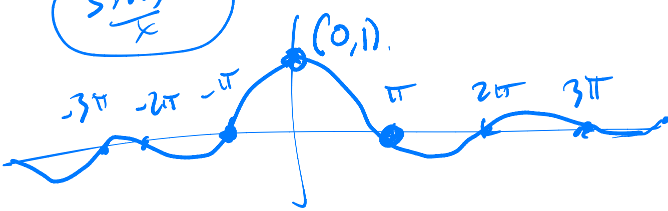
Compound interest ↗ x id. ↑ + identity ↑

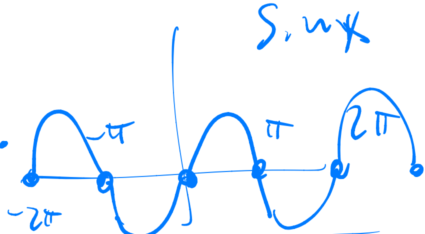
Therefore God exists

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\frac{\sin x}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots$$

$\frac{\sin x}{x}$



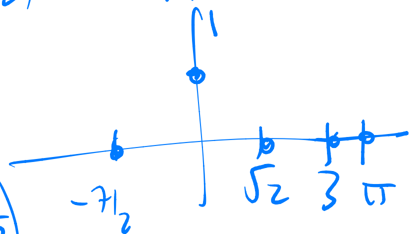


Euler knows: if f is polynomial degree n, $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$.

with roots $\underbrace{\alpha_1, \dots, \alpha_n}$ & $f(0)=1$

Then $f(x) = \left(1-\frac{x}{\alpha_1}\right)\left(1-\frac{x}{\alpha_2}\right)\cdots\left(1-\frac{x}{\alpha_n}\right)$.

α's $= 3, \pi, \sqrt{2}, -\frac{7}{2}$.



$$f(x) = \left(1-\frac{x}{3}\right)\left(1-\frac{x}{\pi}\right)\left(1-\frac{x}{\sqrt{2}}\right)\left(1+\frac{7x}{2}\right).$$

$$= 1 + x\left(-\frac{1}{3} + \frac{-1}{\pi} - \frac{1}{\sqrt{2}} + \frac{7}{2}\right) + \dots + (\dots)x^4.$$

Euler: I think (???)

$$1 \boxed{-\frac{x^2}{3!}} + \frac{x^4}{5!} - \dots$$

$$\frac{\sin x}{x} = \underbrace{\left(1-\frac{x}{\pi}\right)\left(1+\frac{x}{+\pi}\right)}\underbrace{\left(1-\frac{x}{2\pi}\right)\left(1+\frac{x}{+2\pi}\right)}\cdots$$

$$= \left(1-\frac{x^2}{\pi^2}\right)\left(1-\frac{x^2}{4\pi^2}\right)\left(1-\frac{x^2}{9\pi^2}\right)\left(1-\frac{x^2}{16\pi^2}\right)\cdots$$

$$= 1 + 0\cdot x + \underbrace{x^2\left(-\frac{1}{\pi^2} - \frac{1}{4\pi^2} - \frac{1}{9\pi^2} - \frac{1}{16\pi^2} - \dots\right)}$$

$$+ x^3(0) + x^4(\dots) + \dots$$

comparing coefficients in front of x^2, Euler gets:

$$-\frac{1}{\pi^2}\left(1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\dots\right)=-\frac{1}{6}.$$

$$1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\dots=\frac{\pi^2}{6}=1{,}6449\dots$$

$$\underbrace{1+\frac{1}{8}+\frac{1}{27}+\frac{1}{64}+\frac{1}{125}+\dots}=\frac{???}{1{,}20\dots}$$

THE

MATHEMATICAL PRINCIPLES

OF

NATURAL PHILOSOPHY.

DEFINITIONS.

DEFINITION I.

The quantity of matter is the measure of the same, arising from its density and bulk conjunctly.

THUS air of a double density, in a double space, is quadruple in quantity; in a triple space, sextuple in quantity. The same thing is to be understood of snow, and fine dust or powders, that are condensed by compression or liquefaction; and of all bodies that are by any causes whatever differently condensed. I have no regard in this place to a medium, if any such there is, that freely pervades the interstices between the parts of bodies. It is this quantity that I mean hereafter everywhere under the name of body or mass. And the same is known by the weight of each body; for it is proportional to the weight, as I have found by experiments on pendulums, very accurately made, which shall be shewn hereafter.

DEFINITION II.

The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjunctly.

The motion of the whole is the sum of the motions of all the parts; and therefore in a body double in quantity, with equal velocity, the motion is double; with twice the velocity, it is quadruple.

DEFINITION III.

The vis insita, *or innate force of matter, is a power of resisting, by which every body, as much as in it lies, endeavours to persevere in its present state, whether it be of rest, or of moving uniformly forward in a right line.*

This force is ever proportional to the body whose force it is; and differs nothing from the inactivity of the mass, but in our manner of conceiving

AXIOMS, OR LAWS OF MOTION.

LAW I.

Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.

PROJECTILES persevere in their motions, so far as they are not retarded by the resistance of the air, or impelled downwards by the force of gravity. A top, whose parts by their cohesion are perpetually drawn aside from rectilinear motions, does not cease its rotation, otherwise than as it is retarded by the air. The greater bodies of the planets and comets, meeting with less resistance in more free spaces, preserve their motions both progressive and circular for a much longer time.

LAW II.

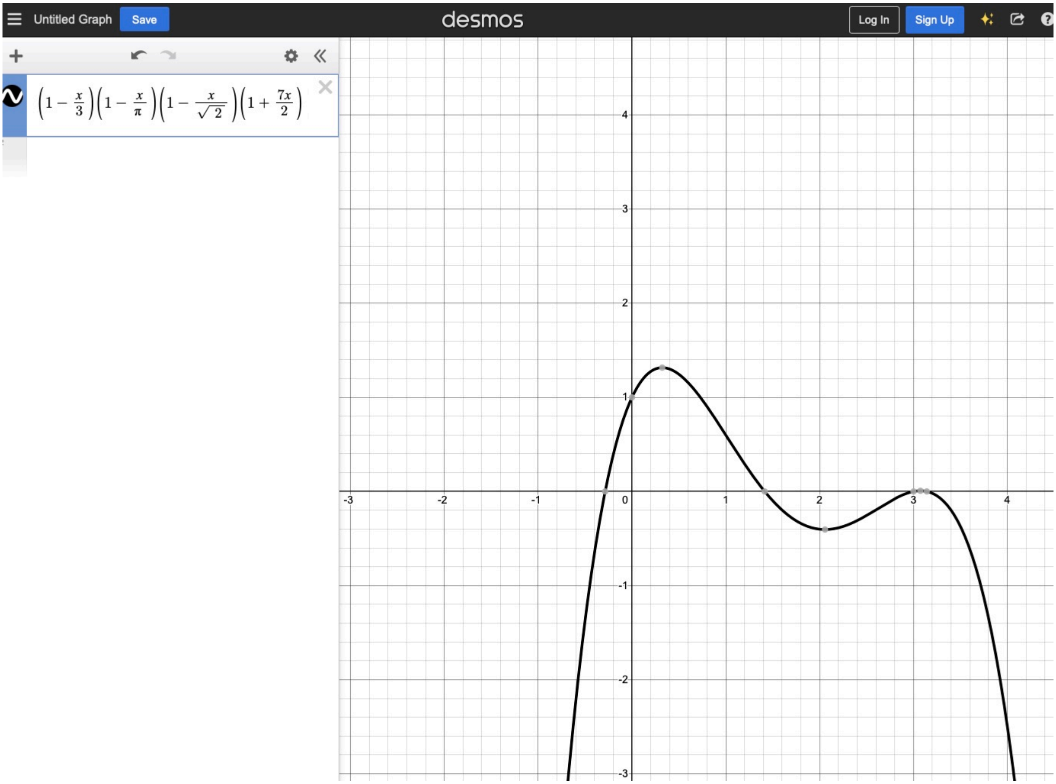
The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.

If any force generates a motion, a double force will generate double the motion, a triple force triple the motion, whether that force be impressed altogether and at once, or gradually and successively. And this motion (being always directed the same way with the generating force), if the body moved before, is added to or subducted from the former motion, according as they directly conspire with or are directly contrary to each other; or obliquely joined, when they are oblique, so as to produce a new motion compounded from the determination of both.

LAW III.

To every action there is always opposed an equal reaction: or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

Whatever draws or presses another is as much drawn or pressed by that other. If you press a stone with your finger, the finger is also pressed by the stone. If a horse draws a stone tied to a rope, the horse (if I may so say) will be equally drawn back towards the stone: for the distended rope, by the same endeavour to relax or unbend itself, will draw the horse as much towards the stone, as it does the stone towards the horse, and will



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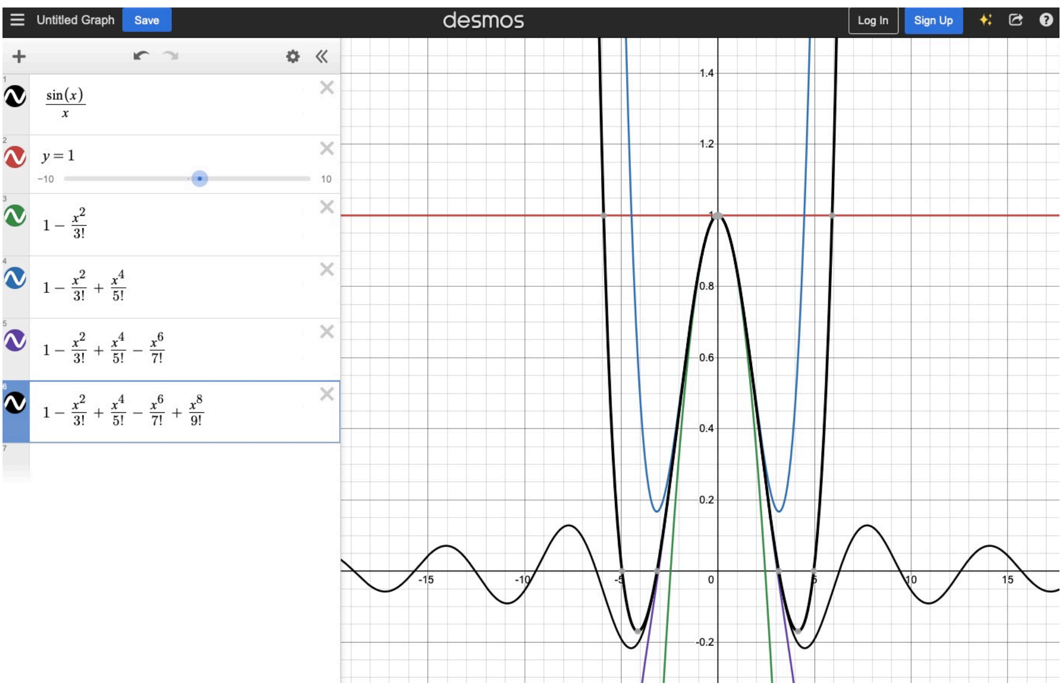
$\left(1-\frac{x}{3}\right)\left(1-\frac{x}{\pi}\right)\left(1-\frac{x}{\sqrt{2}}\right)\left(1+\frac{7x}{2}\right)$



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$\frac{\sin(x)}{x}$
$y=1$
−10
10
$1-\frac{x^2}{3!}$
$1-\frac{x^2}{3!}+\frac{x^4}{5!}$
$1-\frac{x^2}{3!}+\frac{x^4}{5!}-\frac{x^6}{7!}$
$1-\frac{x^2}{3!}+\frac{x^4}{5!}-\frac{x^6}{7!}+\frac{x^8}{9!}$