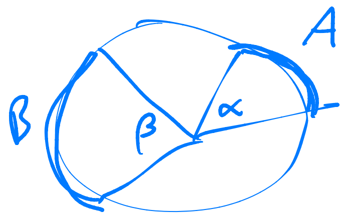
Last time:

Book V: Eudoxus's theory of proportion, fixing Pythagorean (false) axiom of commensurability

Book VI : similarity (AAA) VI.33

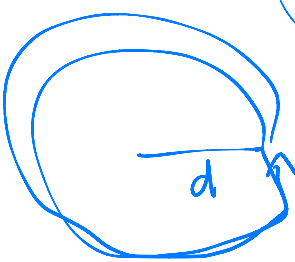
$$\frac{\alpha}{\beta} = \frac{A}{B}.$$



Radian:

distance d. = covers proportion $\frac{d}{2\pi d}$ of circle. $= \frac{1}{2\pi}$

distance d.



Circumference $= \underline{2\pi d}.$

Book VII - IX : number theory
Book X : incommensurability (irrationality)

Book VII Def 1: You are allowed to declare any given length a unit.
Def 2: a whole number = something "measured" by a unit.

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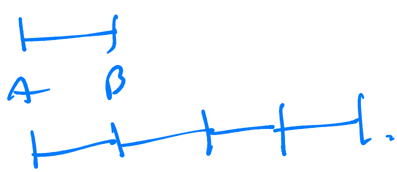
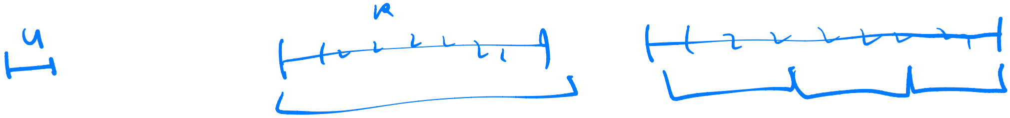


$3 \mid 15$

$3 \cdot 5 = 15.$

Def 3: The number k is a divisor of the number n if k "measures" n, i.e. if there is a whole number m so that k * m = n.



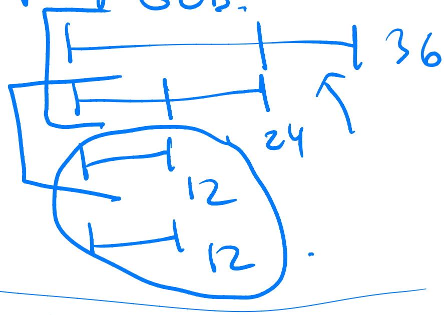
Take n = 8. Divisors of 8: 1, 2, 4, 8. Add up divisors other than 8 itself: 1+2+4 = 7. So 8 is not "perfect".
Take n = 6. Divisors: 1, 2, 3, 6. 1+2+3 = 6. Perfect! (1+2 = 3 = prime).
Take n = 28. Divisors: 1, 2, 4, 7, 14, 28.
1+2+4+7+14 = 28. Perfect!

Book IX.36 (Big boss of Number Theory chapters): If
P = 1+2+4+8+...+2^k
happens to be a prime number P (E.g. 1+2+4 = 7 = prime), then
P * 2^k = N is perfect!!!

Non example: 1+2+4+8 = 15 NOT PRIME. 15 * 8 = 120. Check for yourself: 120 is not perfect.

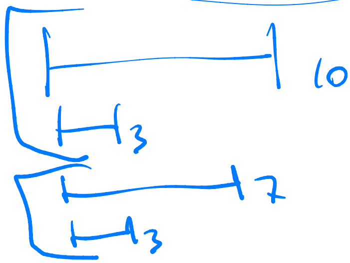
Book VII. Prop 1: Prop 2: Find GCD.

n = 36, m = 24



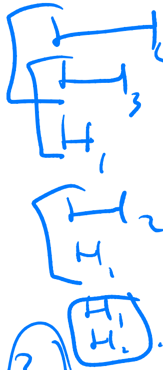
"Euclidean algorithm"

n = 3, m = 10



⇒ 3 & 10 are "relatively prime"

RP, they have no common factor greater than 1



$$\begin{matrix}10\\3\end{matrix} \to \begin{matrix}7\\3\end{matrix} \to \begin{matrix}4\\3\end{matrix} \to \begin{matrix}1\\3\end{matrix} \quad \begin{matrix}3\\1\end{matrix} \to \begin{matrix}2\\1\end{matrix} \to \begin{matrix}1\\1\end{matrix}.$$

$$\begin{matrix}36\\24\end{matrix} \to \begin{matrix}12\\24\end{matrix} \quad \begin{matrix}24\\12\end{matrix} \to \begin{matrix}12\\12\end{matrix}.$$

Book IX Prop 20: Given any collection of prime numbers, there is always more.
Proof. Given a list L of prime numbers: L = {2, 3, 5, ..., P}.
Consider the number N = 2*3*5*...* P + 1.
Claim : N is not divisible by any of the primes in L.
Why? Can't be divisible by 2 because 2*3*5*...* P is even. Add one and it's odd, so not divisible by 2.
By 3? 2*3*5*...* P is divisible by 3. So when I add 1... N can't be divisible by 3.
And so on. N is not divisible by any of the primes in the list.
But any number can be factored (uniquely) into a product of primes.
So for any finite list of primes, there must exist more prime numbers that are not on the list.

8 = 5 +3
12 = 7 + 5
36 = 31 + 5
72 = 67 + 5

38 = 31 + 7

Question: Is every even number at least 4 the sum of two primes?
Goldbach conjecture
Theorem : YES. JUST KIDDING. This is still an UNSOLVED problem in mathematics today!

We do not have a *right* to knowledge! Every time mathematics reveals to us a truth about the universe is a moment we must cherish!

Back to perfect numbers: Euclid says that IF
1+2+4+8+...+2^k = 2^(k+1) - 1 = P
happens to be prime (now they're called "Mersenne primes")
then
N = P * 2^k
is perfect.

Euler 18th century: If N is perfect and even, then it comes from the kind described by Euclid. I.e., N/ 2^k = prime = 2^(k+1) -1.

What about odd perfect numbers? None known !! Conjecture: None exist. OPEN PROBLEM.