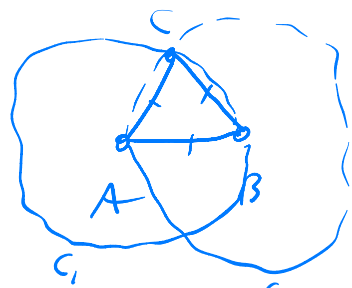
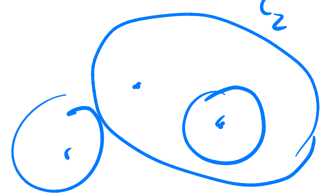
Last time: Euclid's Elements.

Book I: Def, P1–P5,

Proposition 1 : To construct an equilateral triangle on a segment

What	Why
(0) Given $\overline{AB}$	assumption
(1) Draw ⊙ c A r AB $= c_1$	(P3)
(2) Draw ⊙ c B r AB $= c_2$	(P3)
(3) Let C be intersection	$\boxed{????}$
(4) Draw AC	(P1)
(5) Draw BC	(P1)





Def I.15.

$\overline{AB} = \overline{AC}$ (both radii of c_1)

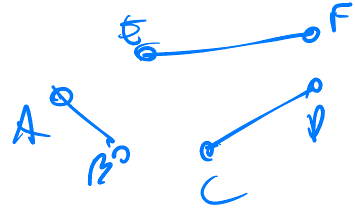
$\overline{AB} = \overline{BC}$ (both radii of c_2)

Claim ΔABC is equilateral,

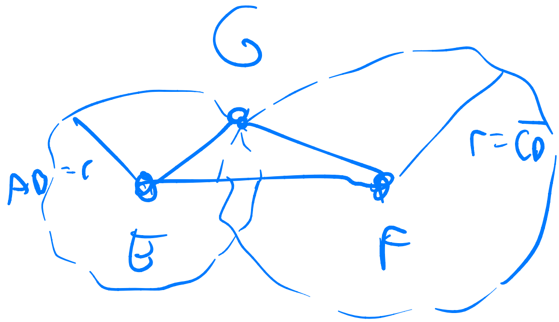
& $\overline{AC} = \overline{BC}$ (CN1).

$\Rightarrow$ Def I.20 is satisfied, QED.

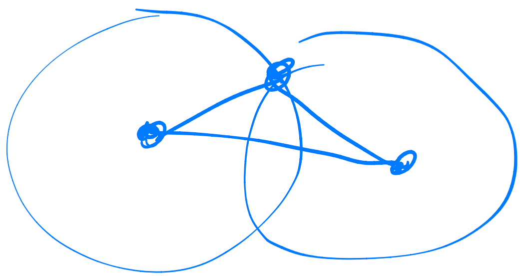
Prop I.22: Given



make a triangle.



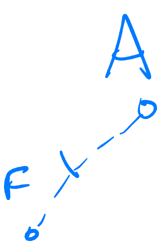
Question that was omitted and is implicitly answered in I.22: when does a pair circles intersect?

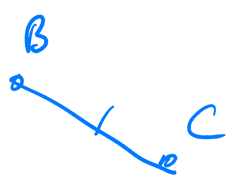


"Triangle inequality": (I.20) the sum of any two sides is greater than the third side

I.2:

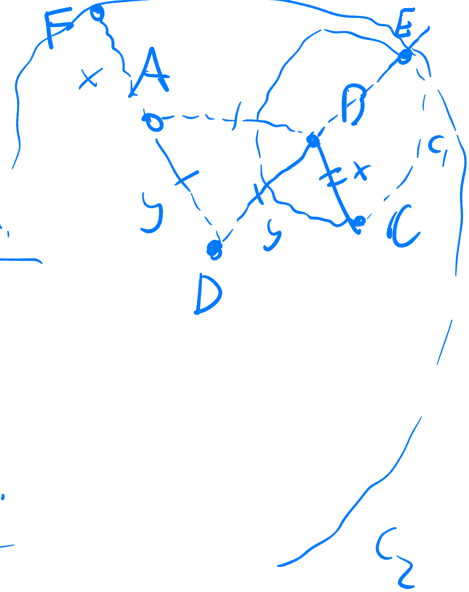
The Euclidean compass is "collapsing" but we can pretend that it locks in place.
Given a point and a segment, you can move the segment to the point.





Given A & $\overline{BC}$, can construct F so that $\overline{AF} = \overline{BC}$

What	Why
(0) Given A & $\overline{BC}$	assumption.
(1) Draw $\overline{AB}$	(P1)
(2) Make equilateral $\triangle ABD$ on AB	(I,1).
(3) Draw ⊙ c B, r BC = c_1	(P3)
(4) Extend BD until it hits c_1, at E	(P2).
(5) Draw ⊙ c D, r DE c_2 =	(P3)
(6) Extend DA to F on c_2	(P2)



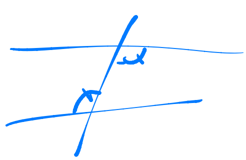
$\overline{DF} = x+y$.

$\Rightarrow \overline{AF} = x$.

F is the desired point.

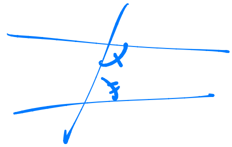
$\overline{AF} = \overline{BC}$

I.27: If



then lines are $\parallel$.

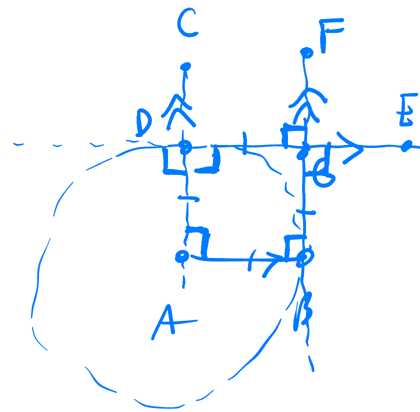
(P5): If



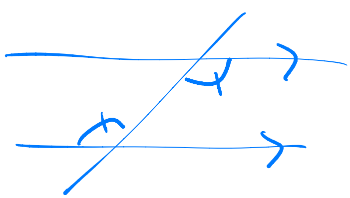
$\gamma + \gamma < \square + \square$

$\Rightarrow$ lines intersect.

Prop I.46: To construct a square on a given segment.



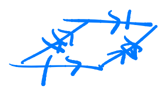
What	Why
(0) Given A, B, $\overline{AB}$	assumption
(1) Make $AC \perp AB$	(I.11)
(2) Let D on AC have $\overline{AD} = \overline{AB}$	(I.3)
(3) Draw $DE \parallel AB$	(I.31)
(4) Draw $BF \parallel AD$	(I.31)
(5) Let G be $BF \cap DE$	~



(I.29)

Claim: ABGD is a square

Pf: ABGD is a parallelogram

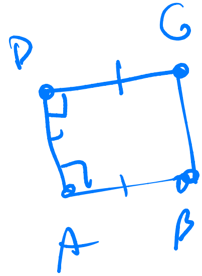


(I.34): opposite sides in a parallelogram are =.

$\overline{AB} = \overline{DG}$, $\overline{AD} = \overline{BG}$.

Now chase angles across differents pairs of parallel lines to show that all the angles are right. QED.

Question: Isn't the following a better construction, and doesn't it NOT rely on drawing parallels (and the parallel postulate)???

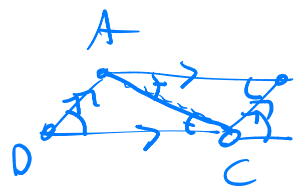


Draw $\perp$ $\overline{AD}=\overline{BA}$

Draw $\perp$ $\overline{DG}=\overline{AD}$

Draw BG ~~Done.~~

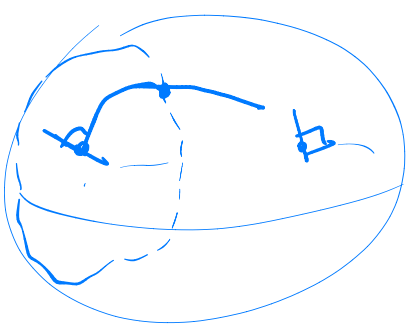
Aside: (I,34)

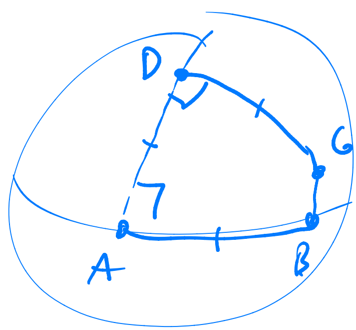


Given ABCD parallelogram. Claim: $\overline{AB}=\overline{DC}$ & $\overline{AD}=\overline{BC}$.

Pf: Draw AC. Then $\triangle ACD \cong \triangle CAB$.

Answer to Question: NOT TRUE unless we use P5.

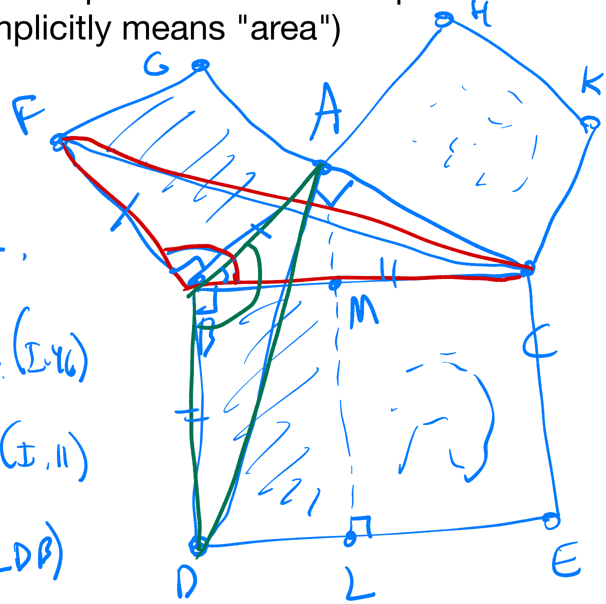




I.47 Pythagorean thm:

In a right triangle, the sum of the squares on sides is equal to the square on the hypotenuse. (Implicitly means "area")

"Windmill proof"



(0) Given $\triangle ABC$, $\angle CAB = \square$.

(1) Construct squares on sides (I.46)

(2) Draw $\perp$ to DE thru A (I.11)

Claim: Area(ADFG) = Area(MLDB)

(3) Draw CF & AD (P1).

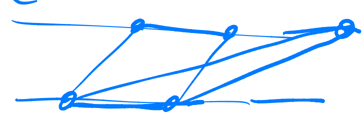
Claim: $\triangle CBF \cong \triangle DBA$.

Apply SAS. $\overline{AB} = \overline{BF}$ (square)

$\overline{BC} = \overline{BD}$ (square)

$\angle CBF = \angle DBA$ Both $90^\circ + \angle ABC$!!

(I.41)

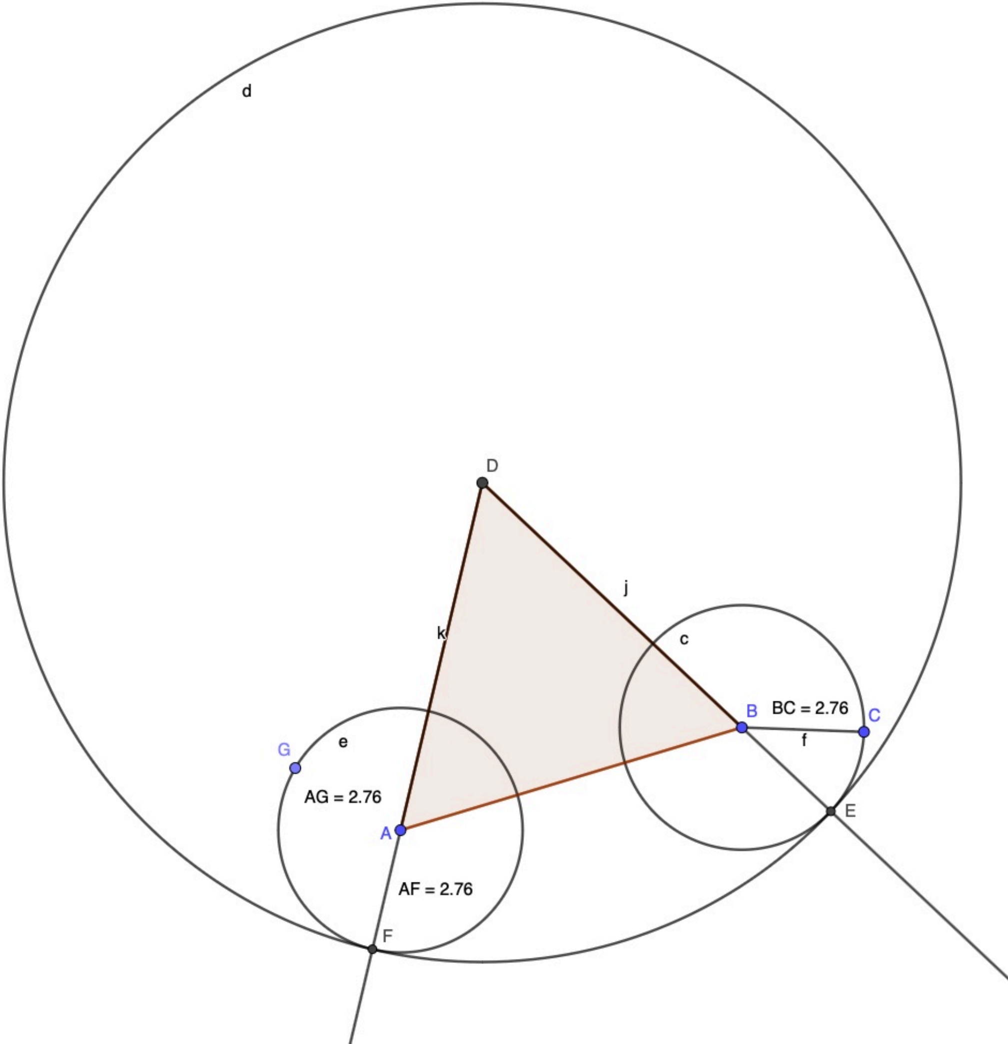


Area(parallelo) = 2 Area(triangle)

So Area(ABFG) $\underset{(I.41)}{=}$ 2 Area($\triangle CBF$)

= 2 Area($\triangle DBA$)

= Area(MLDB) ✓



d
D
j
k
c
B
BC = 2.76
C
f
G
e
AG = 2.76
A
E
AF = 2.76
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