Graph of morning commute
d = distance from home

Average speed (rate of change of distance from home) from 8 to 9 am?

Take a curve \( y = x^2 \)

Equation of line secant to graph of \( y = x^2 \) at \( x = 1 \) & \( x = 3 \):

\[ y = 4x - 3 \]

\[ y - 1 = 4(x - 1) \]

Do same for \( x = 1 \) & \( x = 2 \):

\[ y - 1 = \frac{3}{h} (x - 1) \]

Instead of value at \( h = 0 \), need limit as \( h \to 0 \).

\[ \text{Slope}(h) = \frac{(1+h)^2 - 1^2}{h} = \frac{4h + 2h^2}{h} \]

\[ \text{Slope}(0) = \lim_{h \to 0} \frac{4h + 2h^2}{h} = \lim_{h \to 0} \frac{4 + 2h}{1} = 4 \]
\[ \text{Slope as } h \to 0 \to 2 \]
\[ 2 = \frac{\text{instantaneous rate}}{= \text{slope}} \]
\[ \Rightarrow \text{Equation of line tangent to } y = x^2 \text{ at } x = 1 \]
\[ y - 1 = 2(x - 1) \]

\[ \text{Slope}(h) = \frac{(2+h)^2 - 2^2}{h} \]
\[ \frac{4 + 4h + h^2 - 4}{h} \times 1 \quad \text{as } h \to 0 \to 4 \]

\[ y = \sqrt{x} \quad \text{at } (y, \sqrt{y}) \]

\[ \text{Slope}(u) = \frac{\sqrt{u} - 2}{u - 4} \]

\[ \text{Slope of tangent line to } y = x^2 \text{ at } x = 2. \]

\[ y - 4 = 4(x - 2) \]

\[ \text{Quiz: Work out equation of tangent line to } y = x^2 \text{ at } x = 2. \]
As \( u \to y \),
\[
\lim_{u \to y} \frac{(Ju-2)(Ju+2)}{(u-4)(Ju+2)} = \frac{u-4}{(u-4)(Ju+2)} = \frac{1}{Ju+2}.
\]

If \( u \neq y \),
\[
y - 2 = \frac{1}{y} (x - 4)
\]

Eqn:
\[
y - 2 = \frac{1}{y} (x - 4)
\]

\[
\text{Q: Avg rate of change of } y = \sin x \text{ from } x = 0 \text{ to } x = \frac{\pi}{3}.
\]
\[
\text{Slope} = \frac{\frac{\sqrt{3}}{2}}{\frac{\pi}{3}} = \frac{\sqrt{3} \cdot 3}{\pi} = \frac{3\sqrt{3}}{2\pi}.
\]

Can we figure out instantaneous rate of change of \( y = \sin x \) at \( x = 0 \)?

(\( \theta, \sin \theta \))
Slope(\(\theta\)) = \frac{\sin\theta}{\theta} \quad \text{limit:}\quad \lim_{\theta \to 0} \quad \text{as} \quad \theta \to 0.

\[
\frac{1}{2} \sin\theta \cos\theta < \frac{1}{2} \theta < \frac{1}{2} \frac{\sin\theta}{\cos\theta}
\]

\[
\cos\theta < \frac{\theta}{\sin\theta} < \sec\theta.
\]

\(\theta\) \quad \text{is} \quad \text{squeezed} \quad \text{between} \quad \cos\theta \quad \text{and} \quad \sec\theta.

\[
\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1.
\]