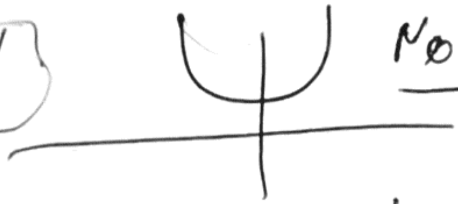
$\mathbb{C} = \{x + iy \mid x, y \in \mathbb{R}\}$.

Why to study calculus / $\mathbb{C}$?

Need $\mathbb{C}$ to solve quadratics?

$x^2+1=0$. $\quad i = \sqrt{-1}$.

$y = x^2+1$

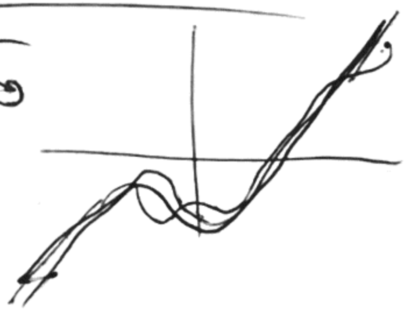


No Solns

People cared about $i=\sqrt{-1}$ before they cared about $\boxed{-1}$

$y = x^3 + 12x - 15$

Cardano Cubic



1495 Paciolli (?)

cubics are as unsolvable as squaring circle?

del Ferro - 1510s

"solves depressed cubic"

$ax^3 + \cancel{bx^2} + cx + d = 0$.

1520s → Fior. goes on attack.

↳ Tartaglia (re)figures out how to solve cubics

Cardano 1530s.

gets Tartaglia to reveal solution.

Ferrari → all cubics → all quartics

Why don't they have negatives?

$x^3 + c = dx^2$.

$x^3 = c + dx^2$

$x^3 \boxed{+} dx^2 = c$

$ax^3+bx^2+cx+d=0$. $\quad a\neq 0 \to a=1$. (make it monic)

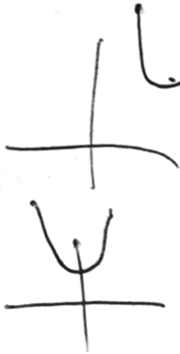
Let $y = x+\frac{b}{3}$

$x = y-\frac{b}{3}$ (depress).

$\underbrace{\left(y-\frac{b}{3}\right)^3} + \underbrace{b\left(y-\frac{b}{3}\right)^2} + c\cdot\left(y-\frac{b}{3}\right)+d=0$.

$y^3 - 3\cdot\frac{b}{3}y^2 + \ldots + by^2 + \ldots = 0$.

$\boxed{y^3+Ay+B=0}$



$ax^2+bx+c=0$. (monic)

$x^2+\frac{b}{a}x+\frac{c}{a}=0$. (depress)

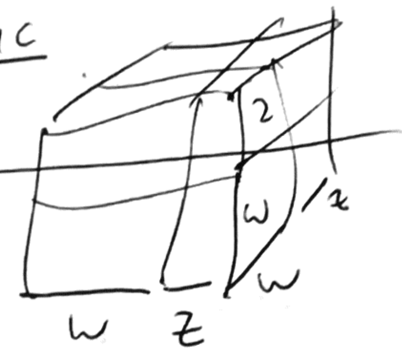
$x = y-\frac{b}{2a}$

$\left(y-\frac{b}{2a}\right)^2 + \frac{b}{a}\left(y-\frac{b}{2a}\right) + \frac{c}{a}=0$.

$y^2 - \cancel{2\frac{b}{2a}y} + \frac{b^2}{4a^2} + \cancel{\frac{b}{a}y} - \frac{b^2}{2a^2}+\frac{c}{a}=0$

$y^2 - \frac{1}{4}\frac{b^2}{a^2}+\frac{4ca}{4a^2}=0$,

$y^2 = \frac{b^2-4ac}{4a^2}$



Idea:

$(w+z)^3$

$= w^3+\underbrace{3w^2z+3wz^2}_{3wz(w+z)}+z^3$.

$\boxed{\underbrace{(w+z)^3}_{y^3}\underbrace{-3wz}_{A}\underbrace{(w+z)}_{y}\underbrace{-z^3-w^3}_{B}=0}$

Need to solve:

$\left.\begin{array}{l} A=-3wz \\ B=-z^3-w^3 \end{array}\right\} \Rightarrow y=z+w$

$w = \frac{A}{-3z}$, $\underbrace{z^3 - \frac{A^3}{27z^3} = -B}$

$$z^6 + Bz^3 - \frac{A^3}{27} = 0.$$

$\boxed{S_3 \text{ solvable}}$

Quadratic in z^3! $\boxed{z^3 + w^3 = -B}$

$$z^3 = \frac{-B + \sqrt{B^2 + \frac{4A^3}{27}}}{2}$$

$$w^3 = \frac{-B - \sqrt{B^2 + \frac{4A^3}{27}}}{2}$$

$$z = \sqrt[3]{-\frac{B}{2} + \sqrt{\frac{B^2}{4} + \frac{A^3}{27}}}$$

$$w = \sqrt[3]{-\frac{B}{2} - \sqrt{\frac{B^2}{4} + \frac{A^3}{27}}}$$

$$y = \sqrt[3]{\cdots} + \sqrt[3]{\cdots}$$

Eg: $x^3 \underbrace{-15}_{A} x \underbrace{-4}_{B} = 0.$

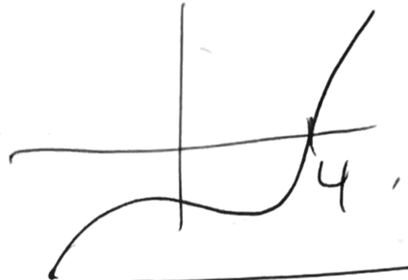
$$\frac{B^2}{4} + \left(\frac{A}{3}\right)^3 = 4 - 125 = -121.$$

$$x = \sqrt[3]{2+11i} + \sqrt[3]{2-11i}$$

Realize: $(2+i)^3 = 8 + 3\cdot 2^2 \cdot i - 3\cdot 2 - i$

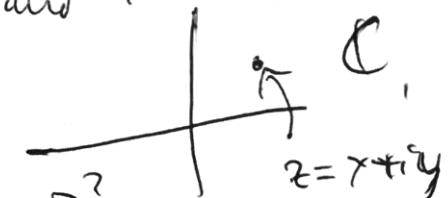
$$= \underbrace{2 + 11i}.$$

$$x = (2+i) + (2-i) = \underline{\underline{4}}.$$



Euler: $\underline{e^{i\theta} = \cos\theta + i\sin\theta}$

Wessel / Argand 1800s Gauss.



Cauchy

$\mathbb{R}^2 \to \mathbb{R}^2$

$f: \mathbb{C} \to \mathbb{C}$.

deriv
integrals.

Ars Magna by Cardano (1545)

HIERONYMI
CARDANI,
ARTIS MAGNÆ,
SIVE
DE REGVLIS ALGEBRAICIS,
LIBER VNVS.

ANDREÆ OSIANDRO
viro eruditiss.
S. P. D.

NIHIL tam animo vnquam versaui, Andrea doctiss. quàm vt eorum, qui de bonis litteris bene merentur, nomina posteritati commendarem. Tum verò præcipuam quandam diligentiam adieci, si tales cum eruditione humanitatem coniunxissent. Quamobrem cùm te non solùm Hebræarum, Græcarum ac Latinarum litterarum scientiam haud mediocrem, sed etiam Mathemeticarum ha-

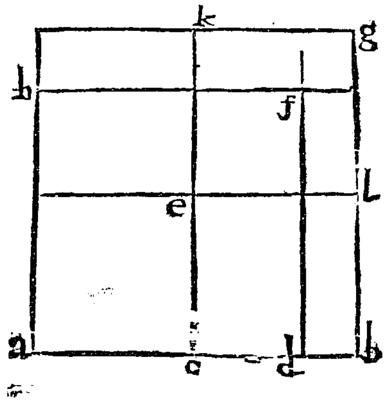
CAPVT XI.

De Cubo & rebus æqualibus Numero.

SCIPIO Ferreus Bononiensis iam annis ab hinc triginta fermè capitulum hoc inuenit, tradidit verò Anthonio Mariæ Florido Veneto, qui cùm in certamen cum Nicolao Tartalea Brixellense aliquando venisset, occasionem dedit, vt Nicolaus inuenerit & ipse, qui cum nobis rogantibus tradidisset, suppressâ demonstratione, freti hoc auxilio, demonstrationem quæsiuimus, eámque in modos, quod difficillimum fuit, redactam sic subiiciemus.

DEMONSTRATIO.

Sit igitur exempli causâ cubus g h, & sexcuplum lateris g h æquale 20. & ponam duos cubos a e & c l, quorum differentia sit 20. ita quod productum a c lateris, in cubo. Iam ergo ventum est necessariò ad Triarios, sit ergo a b diuisa in tres partes, quæ omnes sint ℞. cu. incommensæ, nec in eadē proportione, & constat quòd fient octo genera corporum, vnum quod erit numerus qui constabit ex cubo singularum partium. Cùm enim a c, c d, d b, sint ℞.cu.numerorū, erunt cubi earum numeri: quare & aggregatum eorum numerus. Secundum corpus constabit ex sexcuplo corporis, cuius latera sunt omnes partes scilicet a c, c d, d b, iam



ergo habes nouem corpora. Reliqua decemocto cùm sint tria, & tria æqualia, erunt ergo sex, primum constabit ex c d in triplum quadrati a c, secundum ex b d in triplum quadrati a c, tertium ex a c in triplum quadrati c d, quartum ex b d in triplum quadrati c d, quintum ex a c in triplum quadrati b d, sextum ex c d in triplum quadrati b d, cùm ergo sint septem partes incommensæ in cubo, & tres tantùm in re, cubus non poterit æquari rebus sub aliquo numero. Ostendo modo quod ita sit: nam in superficie a g sunt tria quadrata a e, e f, f g: & sex superficies quarum binæ, & binæ sunt æquales d e e h, & d l h k, & l f f k. At ex a c, c d, d b, in sua quadrata fiunt tres cubi, ex a c verò in f l, f k, idem fit quod ex c d in d l h k, & ex b d in d e, e h, igitur constat de nouem iam corporibus in duo redactis. Dico modò quòd ex vna parte in quadratum alterius fiunt tria corpora, vtpote ex a c in e d, e h, & ex c d in, fiunt tria parallelipeda ex c d in quadratum a c, igitur cùm binæ quantitates residuæ multiplicentur in quadratum tertiæ fient sex aggregata ex tribus parallelipedis, omnia igitnr viginti septem reducta ad octo.

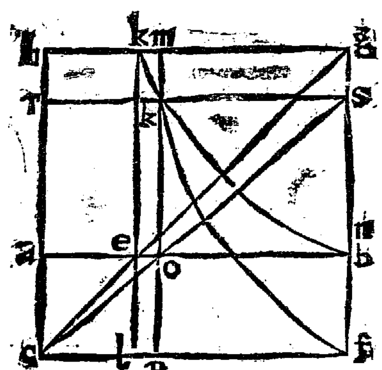
Per 43. primi Elem

CAPVT XII.

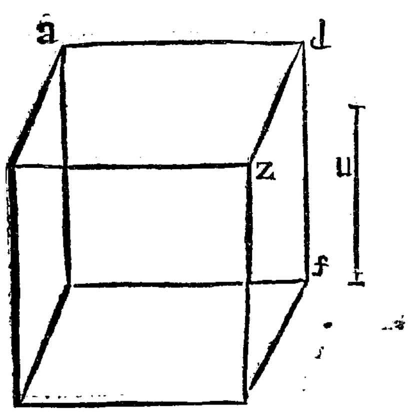
De modo demonstrandi geometricè æstimationem cubi & numeri æqualium quadratis.

SINT quadrata duodecim æqualia cubo, & centum nonaginta duobus numero, gratiâ exempli, & constat ex supradictis, quòd si numerus esset maior ducentis quinquaginta sex, quòd propositum esset falsum, & si esset ipse numerus ducenti quinquaginta sex, quòd latus cubi esset octo seu bes eiusdem numeri quadratorum, & ideò proposuimus numerum illo minorem. Et ex eisdem constat quod si numerus esset dimidium maximi numeri, scilicet centum viginti octo, quòd res esset tertia pars numeri quadratorum propositorum, quia proportio quadrati bessis ad quadratum trientis est velut bessis ad id quod prouenit diuiso centum viginti octo solido proposito per quadratum bessis, quod est sexaginta quatuor, exit enim duo qui est quarta pars octo, vt sexdecim quadratum quatuor, trientis est quarta pars sexaginta quatuor quadrati octo bessis numeri quadratorum propositi. Nos ergo sumpsimus alium numerum ab his vt dixi. Proponatur ergo corpus solidum d q t z rectilineum & æquidistantium laterum ac superficierum, cuius ima superficies sit d q t quadrata, & sit totum solidum centum nonaginta duo, scilicet numerus propositus, & eius altitudo sit linea d z, & sit a b data duodecim æqualis, scilicet numero quadratorum proposito, & diuisa ita vt b e sit dupla ad e a. Et duabus c b &

Prop. 27:



d q subtendatur linea quædam u, & sit d z ad a c; vt e b ad u, erit ergo quadrati e b ad quadratum q t, vt e b ad u; quare vt d z ad a c: igitur solidum quod sub a c & quadrato e b æquale solido d q t z, propositum igitur est sic diuidere a b, vt solidum ex vna parte in quadratum alterius sit æ-



quale solido ex a c in quadratum e b. Et hoc nos docet facere Eutocius Asca-