Recall: P-value: H_0, H_1, E.g.: $C = \{\bar{x} > k\}$, $P(C; H_0) = \alpha$.

Def: P-value (probability): least $\underline{\alpha}$ for which H_0 is rejected.

E.g.: 100 tires last avg $\bar{x} = 21{,}819$ mi, Known $\sigma = 1295$ mi

Test $H_0: \mu_0 = 22{,}000$ vs $H_1: \mu < 22{,}000$.

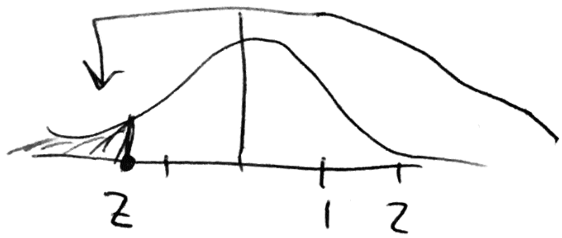
~~at the level of significance of 5%~~ $\alpha = 0.05$

Underlying assumption: tire duration ~ normal

Assume H_0: $\bar{X} \sim N(\mu_0, \frac{\sigma^2}{n})$ $\qquad C(\bar{x} < k)$.

observed z-value: / z-score

$$z = \frac{\bar{X} - \mu_0}{\sqrt{\sigma^2/n}} = \frac{21819 - 22000}{1295/\sqrt{100}}$$



$= -1.4 = -z_\alpha$.

$\text{CDF} = \Phi(-1.4) = 0.08 = \text{P-val}$

E.g.: Ribbon designed to withhold 185 lbs force.

Quality control tested $n = 5$ ribbons, broke at:

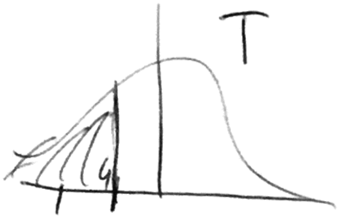
171.6, 191.8, 178.3, 184.9, 189.1, lbs.

sample mean $\bar{x} = 183.1$. sample variance $S = 8.2$.

Test $H_0: \mu_0 = 185$ vs $H_1: \mu_0 < 185$.

$$T \cancel{X} = \frac{\bar{X} - \mu_0}{\sqrt{s^2/n}} = \frac{\bar{X} - \mu_0}{\sqrt{\sigma^2/n}} \cdot \sqrt{\frac{\cancel{\sigma^2} \cancel{(n-1)}}{\cancel{(n-1)} s^2}}$$

$$= Z \cdot \sqrt{\frac{n-1}{Y}}$$



$$T = \frac{183.1 - 185}{8.2/\sqrt{5}} = -0.51 = \text{T-value with}$$

4-deg freedom.

CDF(T-dist 4 degrees, -0.51) = 0.31 = p-value.

At what T-value would I get <u>nervous</u>?

say $\alpha = 0.05$. $-t_{\alpha, 4} = -2.132$.

<u>E.g. 8</u> Brands A & B test for nicotine content.

$n_A = 50$, $\bar{X}_A = 2.61$ mg, Know: $\sigma_A = 0.12$ mg

$n_B = 40$, $\bar{X}_B = 2.38$ mg, " $\sigma_B = 0.14$ mg.

Test $H_0: \mu_A - \mu_B = 0.2$mg. $H_1: \mu_A - \mu_B \neq 0.2$.

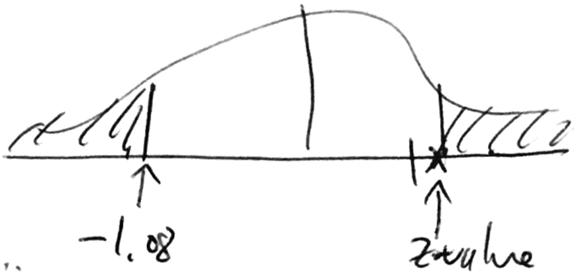
$$\bar{X}_A \sim N\left(\mu_A, \frac{\sigma_A^2}{n_A}\right), \quad \bar{X}_B \sim \dots$$

Assuming H_0,

$$Z = \frac{(\bar{X}_A - \mu_A) - (\bar{X}_B - \mu_B)}{\sqrt{\frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}}} = \frac{2.61 - 2.38 - \overbrace{0.2}^{-(\mu_A - \mu_B)}}{\sqrt{\frac{0.12^2}{50} + \frac{0.14^2}{40}}}$$

z-value

$Z = 1.08$



p-value $= 2\Phi(-1.08) = 0.28..$

What if σ's unknown, $S_A = 0.12$, $S_B = 0.14$.

Variant 1: not equal, maybe we can assume that $\sigma_A = \sigma_B = \sigma$ ~~$= 0.13$~~.

$$T_{n_A+n_B-2} = \frac{(\bar{X}_A - \mu_A) - (\bar{X}_B - \mu_B)}{\sqrt{\frac{\cancel{\sigma^2}}{n_A} + \frac{\cancel{\sigma^2}}{n_B}}} \cdot \sqrt{\frac{n_A + n_B - 2}{\frac{(n_A-1)S_A^2}{\cancel{\sigma^2}} + \frac{(n_B-1)S_B^2}{\cancel{\sigma^2}}}}$$

Assume H_0

$$= \frac{2.61 - 2.38 - .2}{\sqrt{\frac{1}{50} + \frac{1}{40}}} \sqrt{\frac{88}{49 \cdot (0.12)^2 + 39 \cdot (0.14)^2}}$$

$$= 1.094$$

p-value $= 2 \cdot CDF(T_{88}, -1.094) = 0.274$.

If drop assumption $\sigma_A = \sigma_B$, replace it with Approximation: CLT ~~has no~~ applies & $S_A \to \sigma_A$.
Because $n_A = 50$, $n_B = 40$. Use Z-test.

Eg.: Paint cans. $n_A = 4$, $\bar{X}_A = 546$, $S_A = 31$.

$n_B = 4$, $\bar{X}_B = 492$, $S_B = 26$.

Test $H_0: \mu_A = \mu_B$, $H_1: \mu_A > \mu_B$. $\boxed{\text{Assume } \sigma_A = \sigma_B = \sigma.}$

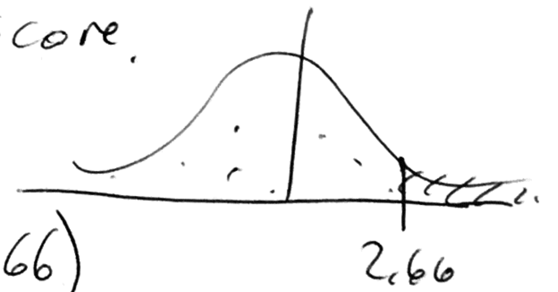
$$T = \frac{(\bar{X}_A - \mu_A) - (\bar{X}_B - \mu_B)}{\sqrt{\frac{\sigma^2}{n_A} + \frac{\sigma^2}{n_B}}} \cdot \sqrt{\frac{n_A + n_B - 2}{\frac{(n_A-1)S_A^2}{\sigma^2} + \frac{(n_B-1)S_B^2}{\sigma^2}}}$$

$$= \frac{546 - 492 - 0}{\sqrt{\frac{1}{4} + \frac{1}{4}}} \sqrt{\frac{6}{3 \cdot 31^2 + 3 \cdot 26^2}}$$

$T_6 = 2.66 \leftarrow$ t-score.

p-value $= 1 - \mathrm{CDF}(T_6, 2.66)$

$= 0.0185$.



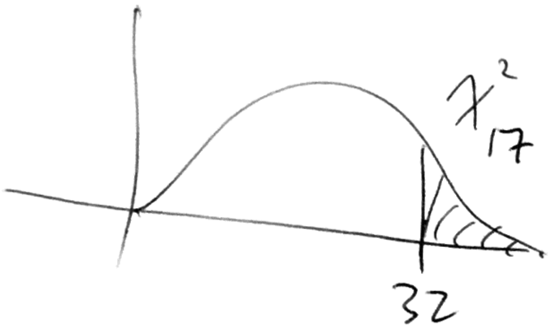
E.g.: $n=18$. $S^2=0.68$. Spec calls for $\sigma^2 \le 0.36$.

Test $H_0: \sigma^2 \le 0.36$ vs $H_1: \sigma^2 > 0.36$.

Assume $\sigma^2 = 0.36$.

$$\chi^2 = \frac{(n-1)\,S^2}{\sigma^2} = \frac{17 \cdot 0.68}{0.36} = 32.11.$$

χ^2-value



$$\text{p-val} = 1 - \mathrm{CDF}(\chi^2_{17}, 32.11)$$
$$= 0.01458.$$
