<u>Recall:</u> Hippasus's proof that $\sqrt{2} \notin \mathbb{Q}$.

<u>Thm:</u> $\sqrt{3} \notin \mathbb{Q}$.

<u>pf:</u> Assume $\sqrt{3} = \frac{p}{q}$ for some $p, q \in \mathbb{N}$ and $\gcd(p,q)=1$.

Then $3 = \frac{p^2}{q^2} \Rightarrow p^2 = 3q^2$;

$\gcd(36,24) = 12$

<u>Fact:</u> $36 = 2^2 \cdot 3^2$

$$36 \to 6, 6; \quad 6 \to 3, 2; \quad 6 \to 3, 2$$

<u>Fact:</u> Every positive integer can be decomposed as a product of <u>primes</u> & uniquely so.

$\Rightarrow 3 \mid p^2$. "divides": ie p^2 is a multiple of 3. $(p^2 = 3 \cdot q^2)$.

If $3 \nmid p \Rightarrow 3 \nmid p^2$.

So $3 \mid p \Rightarrow p = 3\cdot k$, for some $k \in \mathbb{N}$.

So $(3k)^2 = p^2 = 3\cdot q^2 = 9k^2 \Rightarrow q^2 = 3k^2$.

$$\boxed{\underbrace{3(3k^2)}_{=p^2} = 3\cdot q^2}$$

So $3 \mid q^2 \Rightarrow 3 \mid q$.

But $1 = \gcd(p,q) \geq 3$. (Contradiction)

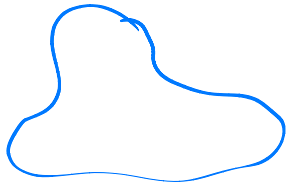
Challenge: Find a geometric argument for $\sqrt{3} \notin \mathbb{Q}$.

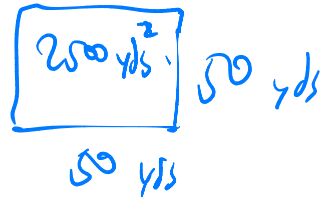
First Great Theorem: <u>Quadrature</u> (of the <u>lune</u>).

Hippocrates of Chios. ~ 400 BC.
(not Hippocratic Oath).

<u>Def</u>: <u>Quadrature</u> = to square something
= "to understand it".
= to construct a square with same area.

To properly tax an area, need to determine

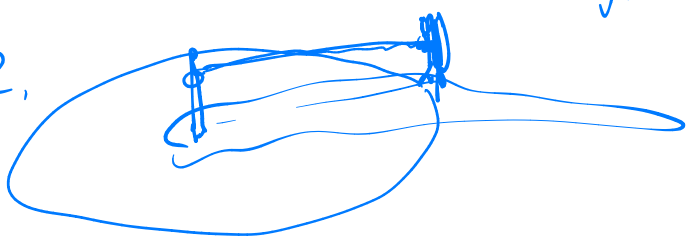




<u>Def</u>: A shape is <u>quadrable</u> if it is possible to <u>construct</u> a square of the same area.

with straightedge & compass.

<u>Tools of surveyors</u>:
- Stakes to plant in ground.
- rope.

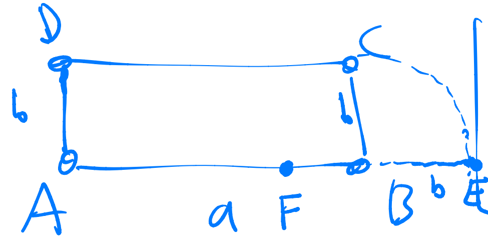


<u>Plato</u>: Objects available to Earthly geometers that best approximate Heavenly geometry are:

- <u>compass</u>.
- ~~ruler~~ <u>straightedge</u> (not allowed to mark length).

<u>Thm</u>: The rectangle is <u>quadrable</u>.

pf:

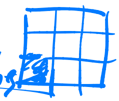


A sides $\left(\frac{1}{3}\,yd\right)^2 = \frac{1}{9}\,yd^2$

$= 1\,ft^2$

$= (12'')^2$

$= 144$ sq inches

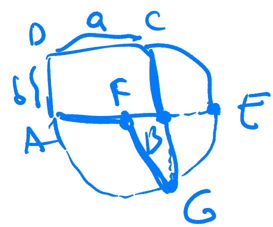


Given rect ABCD. (Area $= a \cdot b$. Need to construct a square with side length $\sqrt{ab}$!).

① Draw $\odot$ c B r $\overline{BC}$.

② Extend AB to intersect $\odot$ at E

Note: $\overline{AE} = a+b$.



③ Bisect $\overline{AE}$ at F.

④ Draw $\odot$ c F r $\overline{AF}$.

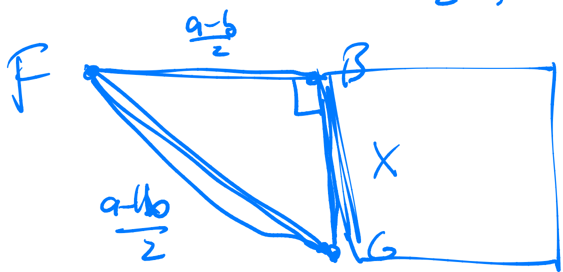
⑤ Extend BC to intersect $\odot$ c F at G. Look at $\triangle BFG$

Look at $\overline{FG} = \overline{AF} = \frac{a+b}{2}$.

Look at $\overline{BF} = \overline{FE} - \overline{BE}$

$= \frac{a+b}{2} - b$

$= \frac{a-b}{2}$.



$$x^2 + \left(\frac{a-b}{2}\right)^2 = \left(\frac{a+b}{2}\right)^2$$

$$x^2 + \cancel{\frac{a^2}{4}} - \frac{ab}{2} + \cancel{\frac{b^2}{4}} = \cancel{\frac{a^2}{4}} + \frac{ab}{2} + \cancel{\frac{b^2}{4}}$$

$$x^2 = ab \;\Rightarrow\; x = \sqrt{ab}$$