Cindy's Solution: $\frac{4}{17} = \frac{1}{6} + \frac{1}{510} + \frac{1}{15}$.

Last time: Pythagoreans ~550BCE

geometry, arithmetic, whole numbers, harmony (music, beginnings of Western), ratios.

Air particles

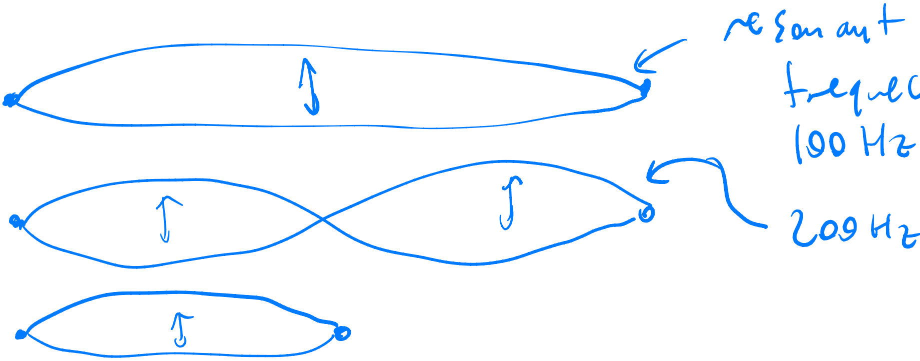
1 atm ≈ 15 psi

At Mariana Trench: 15,000 psi

"Sound" pressure waves of air molecules

audible range of frequency is 20 Hz → 20,000 Hz.
→ 15,000 Hz.

1 Hz = 1 beat per second



Pythagoreans observed: 2x Frequency → "Same" Sound

"octave" = 8 notes up major scale.

Hz:

3×3 $\quad$ $5\times 3 = 3\times 5$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32

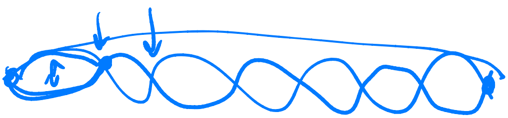
C C G C E G Bb C D E F# G A Bb B C C# D D# E F F# F## G A B B‡ C

"fifth" "third" "Dominant seventh"

diatonic

chromatic scale

microtones.



Pythagoreans "worshipped" whole number ratios.

Cycle of Fifths. $\frac{3}{2}\times$ frequency: 5th.

$\times 3/2$

C → G → D → A → E → B → F# → C# → Ab → Eb → Bb → F → C

1 $\quad$ C $\quad$ C $\quad$ C $\quad$ C $\quad$ C $\quad$ C $\quad$ $\left(\frac{3}{2}\right)^{12}$

$\times 2$ $\quad$ 2^7.

2^7 = 7 octaves = 128

$= \left(\frac{3}{2}\right)^{12}$ = 12 "fifths" = 129.7... Pythagorean Comma.

Theorem: A piano does not exist!

(If it has perfect octaves & perfect fifths).

1500s: Equal temperament tuning.

<u>King</u>: Octave <u>Queen</u>: <u>Semitone</u>.

C $\xrightarrow{\times f}$ C# → D → D# → E $\xrightarrow{\times f}$ F → F# → G → G# → A → A# → B → C

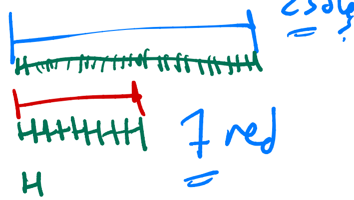
$$f^{12} = 2$$

$$f = \sqrt[12]{2} = 2^{1/12}$$

$$2^{7/12} = f^7 \approx 1.498. \quad \text{Want: } \frac{3}{2} = 1.5$$

C → G.

Pythagorean Axiom of Commensurability:

Given two lengths, 

$\exists$ smaller length measuring both

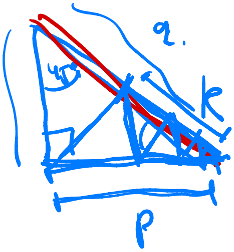
i.e. All ratios of lengths are ratios of whole numbers.

<u>Use this</u>: similar triangles

$$\frac{b}{a} = \frac{s}{r}.$$

Built deep theory of geometry using this axiom everywhere.

Hippasus: look at Isosceles right triangle.



Axiom ⇒ there is a length (unit)

there is a length (unit) H

that measures both side & hypotenuse.

that measures both the side and hypotenuse

Say p units = side & q units = hypot.

Idea: Try to figure out p & q!

$$p^2 + p^2 = q^2 = 2\cdot p^2 = \text{even}.$$

$$\Rightarrow q = \text{even} \Rightarrow q = 2\cdot k.$$

So $\exists$ smaller right triangle, with sides & hypotenuse measured by H. unit.

Hippasus realizes: he can make arbitrarily small isosceles

Hippasus realizes: he can make arbitrarily small Isosc right triangles, all measured by same unit!

triangles, all measured by the same unit. Impossible!

Impossible! Eventually get less than unit!

Eventually get a length that's less than the unit.

Discovered: $\sqrt{2} \notin \mathbb{Q}$ = rational numbers.