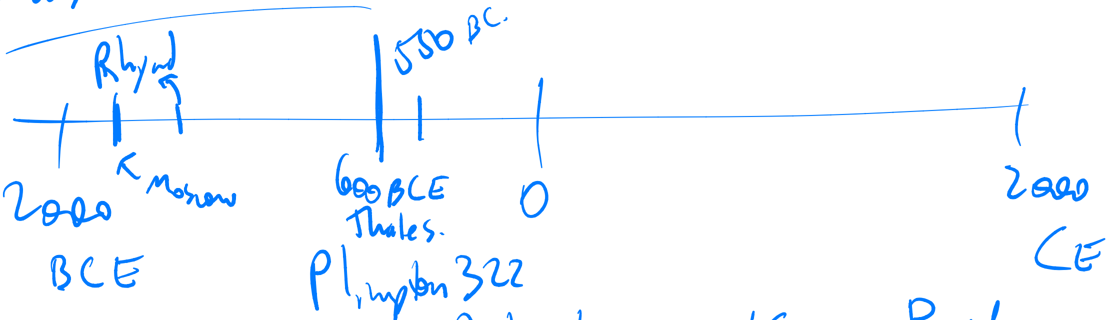
Last time: Pyth.

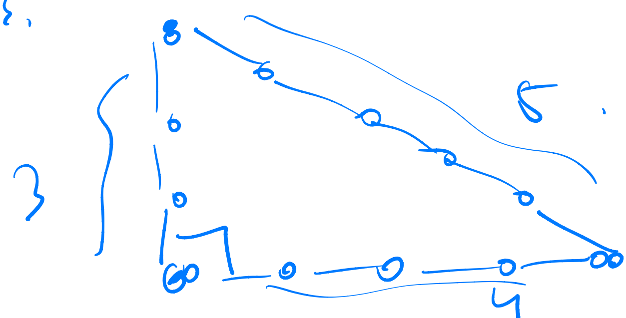


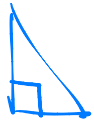
YBC 7289. Babylonians knew Pythag Thm. Long before Pythagoreans.

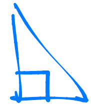
Egyptian: Rhind Papyrus & Moscow. 1550 BCE 1850 BCE

British Museum.

Carry loop of rope, 12 equally spaced knots.



Pyth Thm: If  $\Rightarrow a^2+b^2=c^3$.

Converse Pyth Thm: If $a^2+b^2=c^2 \Rightarrow$ .

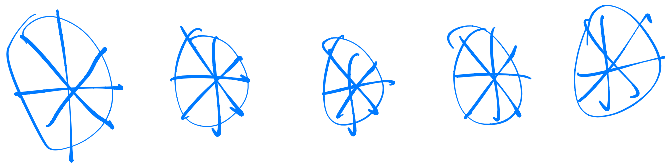
Vinculum: $0.2\overline{345}$ ← vinculum.

$\overline{1+2i} = 1-2i$. $\sqrt{25+3}$

$\frac{7}{12}$, $\overline{|||} = \frac{1}{3}$.

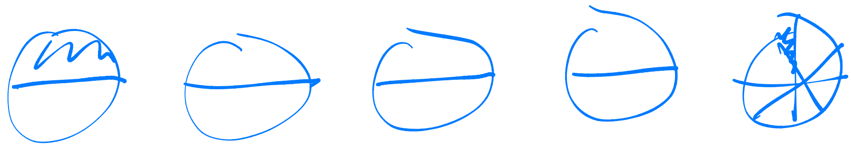
Egyptian (unit) fractions: Expressing fractions using only <u>unit</u> fractions (numerator = 1).

<u>Problem:</u> Job pays 5 loaves. 8 workers do it. How to split?



5 slices $\left(\frac{1}{8}\right)$ each.

$\frac{5}{8}$ = unit fractions? $= \frac{1}{2} + \frac{1}{8}$.



Erdős–Straus Conjecture (1948)

$\forall\, n \geq 2,\ \exists\, x, y, z \in \mathbb{N}$ so that

"for all"   natural number   "there exist"   "in"   "natural #'s" $\{0, 1, 2, 3, \ldots\}$.

$$\frac{4}{n} = \underbrace{\frac{1}{x} + \frac{1}{y} + \frac{1}{z}}$$

$\boxed{n=17}$

$$\frac{4}{17} = \frac{1}{\ } + \frac{1}{\ } + \frac{1}{\ }$$

Exercise!

??

Answer: 4/17 = 1/6 + 1/17 + 1/102

$\boxed{n=8}$

$$\frac{4}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{4}.$$

$(4, 8, 8) = (x, y, z)$.

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=1$$

YBC 7289: computed $\sqrt{2}$ to $\underline{10^{-6}}$.

"Post-engineering, pre-math" result.

Too precise for any real-world application.

For comparison: Construct 1 mi road. If you get things right to within 10^{-6} mi, how big is that?

$1 \text{ mi} = 5280' = 63360''$.

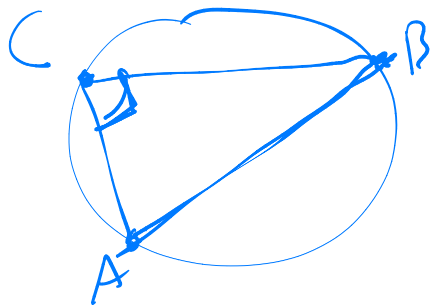
$10^{-6} \text{ mi} = 63360/10^{6}\,'' = 0.06''$.

Why "pre-math": No proof of claimed accuracy.

Earliest known (to me, today) mathematical theorem (with proof) due to: Thales, ~600 BCE

geogebra.org.

Take diameter to circle & random pt on circle.



Q: $\measuredangle C = ?$

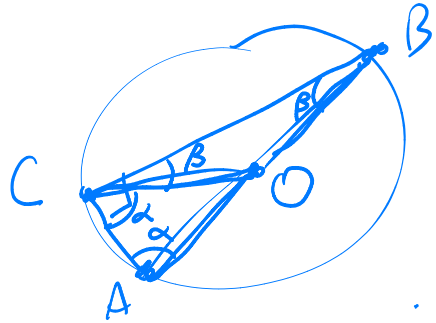
Thm: $\measuredangle C = \llcorner$

Tribunal (England) = 3 judges. are "experts" on determining which FACTS are true

<u>US</u>: <u>jury</u> determines FACTS.

judge is expert on <u>law</u>.

<u>Thales's Thm</u>: If $\overline{AB}$ diameter of circle & C is on circle, then



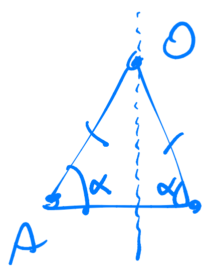
$\measuredangle C = \measuredangle ACB = \square$
$= 90°$

<u>pf</u>: <u>Idea</u>: Draw $\overline{OC}$, where

O = center

$\overline{OA}$ = radius = $\overline{OB} = \overline{OC}$.

Look at ΔOAC



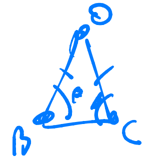
← Isosceles

$\Rightarrow \measuredangle OAC = \measuredangle OCA = \alpha$ ← FACT needs to be established

FACT needs to be established

Similarly, $\triangle OBC$ is isosceles

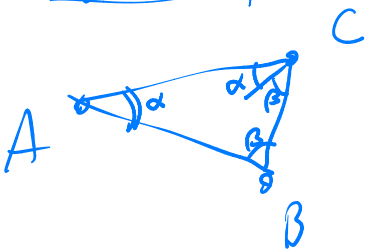
$\Rightarrow \angle OBC = \angle OCB = \beta$



Idea: Sum of angles in $\triangle = 2\square$

$= 180°$.

(Requires proof).



$$\angle A + \angle B + \angle C = 180$$
$$= \alpha + \beta + (\alpha + \beta)$$
$$= 2(\alpha + \beta).$$

$\Rightarrow \alpha + \beta = 90° = \angle C.$

QED.

To establish some facts need other facts, & they need to be established, ...

Pythagoreans (led by Pythagoras), ~550 BC Need to have AXIOMS, "self-evident" truths.

"We hold these truths to be self-evident"

Notice leap from:

→ TRUTHS – YBC.

→ (TRUTH, PROOF) – Thales.

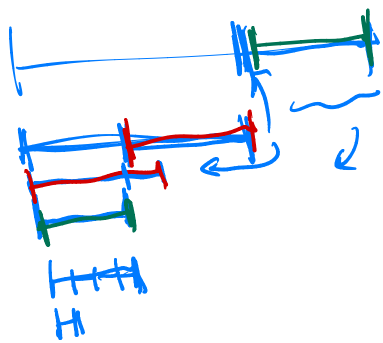
→ (TRUTH, AXIOMS, PROOF) – Pythagoras.

Pythagorean Axiom (of Commensurability):

commensurability

Given two sticks

Break longer one where it sticks out



Do it again, until you stop.

Axiom: Given two sticks, ∃ smaller stick "measures" both evenly.

Axiom: Given two sticks, there exists a smaller stick that "measures" both evenly.