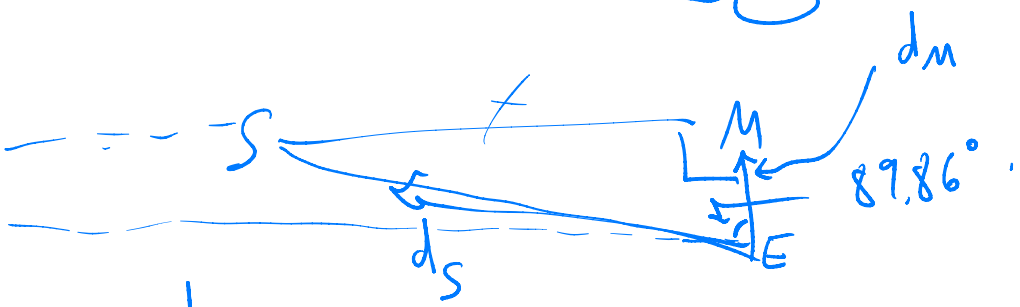
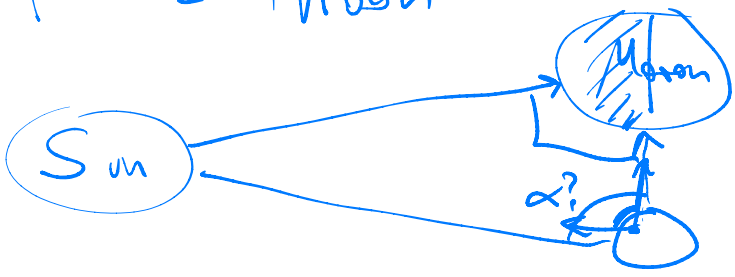


Overview:

- algebra
- geom ✓
- trig ←
- calc

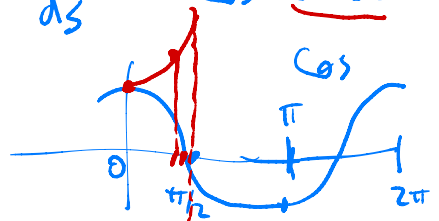
$$\frac{\text{diam}(M)}{\text{dist}(M)} = \frac{\text{diam}(S)}{\text{dist}(S)} = 0.008.$$

A + 1/2 Moon



$$\frac{dm}{ds} = \cos 89.86^\circ$$

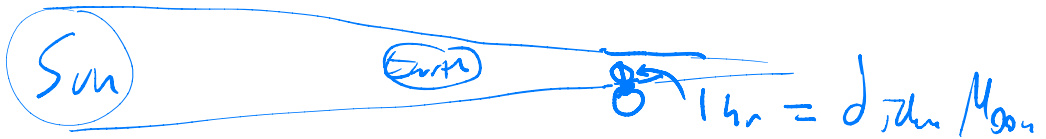
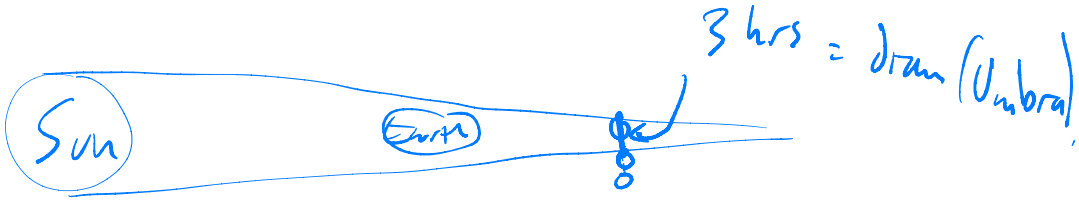
$$ds = dm \cdot \frac{\sec 89.86^\circ}{\approx 400}$$



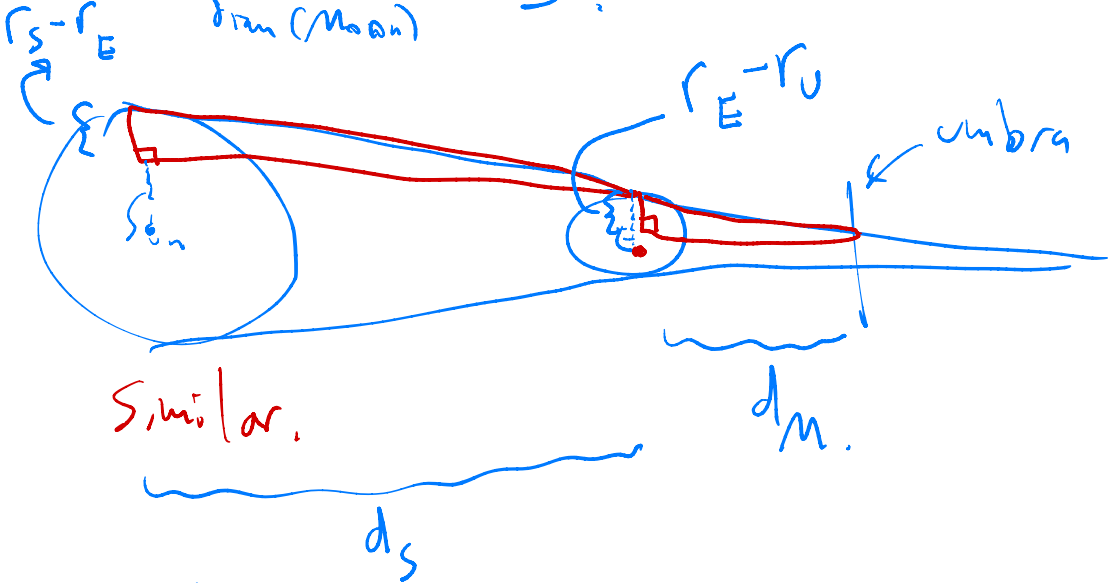
$$\sec = \frac{1}{\cos}$$

Aristachus computed: 87 degrees, got sec = 20

Truth: 89.86 degrees, sec = 400



$$\frac{\text{diam (Umbra)}}{\text{diam (Moon)}} = 3$$



$$\frac{d_m}{d_s} = \frac{r_E - r_U}{r_s - r_E}$$

$$\frac{r_m}{r_s} \stackrel{=}{=} \frac{r_E - 3r_m}{r_s - r_E}$$

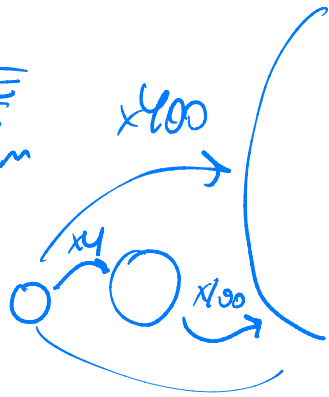
$$\frac{r_s - r_E}{r_s} = \frac{r_E - 3r_m}{r_m}$$

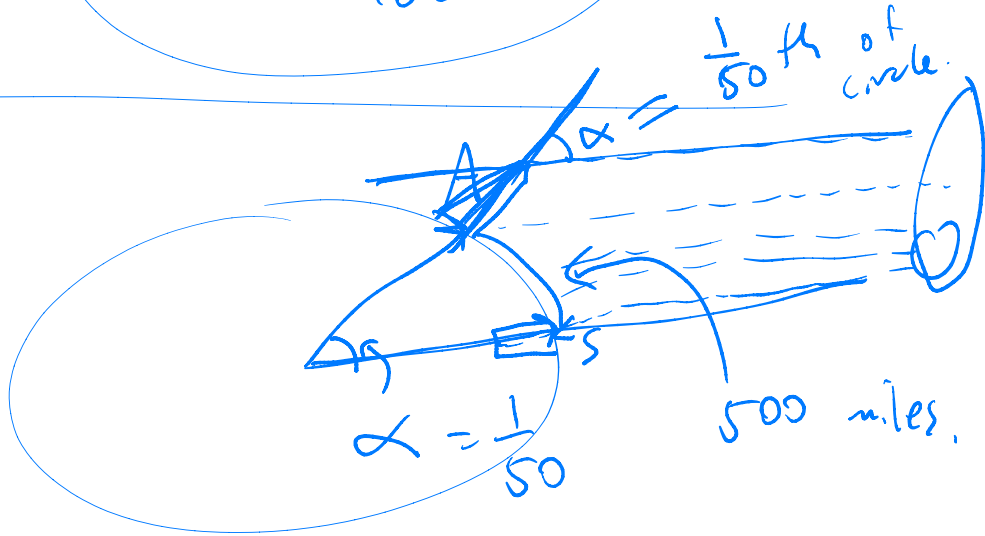
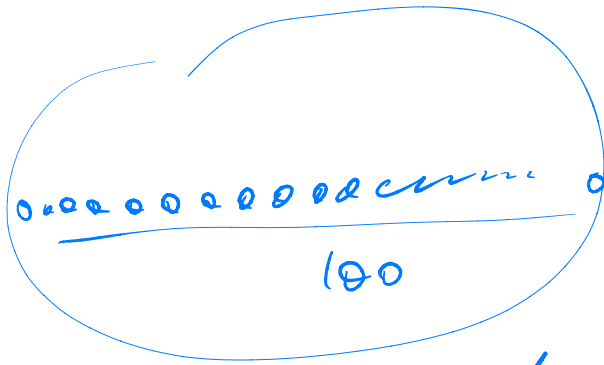
$$\frac{r_E}{400 r_m} = \frac{r_E}{r_m} - 3$$

$$4 = \left(1 + \frac{r_E}{r_m}\right) \frac{r_E}{r_m}$$

$$r_E = 4 r_m$$

$$r_s = 100 r_E$$





circumference (Earth) = $500 * 50 = 25\ 000$ mi

Diameter (Earth) = 8 000 mi

Diameter (Moon) = 2 000 mi

Diameter (Sun) = 800 000 mi

dist(Moon) = 250 000 mi

dist(Sun) = 100 000 000 mi

$$\frac{\text{diam } M}{\text{dist } M} = 0,008$$

$$= \frac{8}{1000}$$

$$= \frac{2}{250}$$

The Birth of Trigonometry: Measuring the Heavens

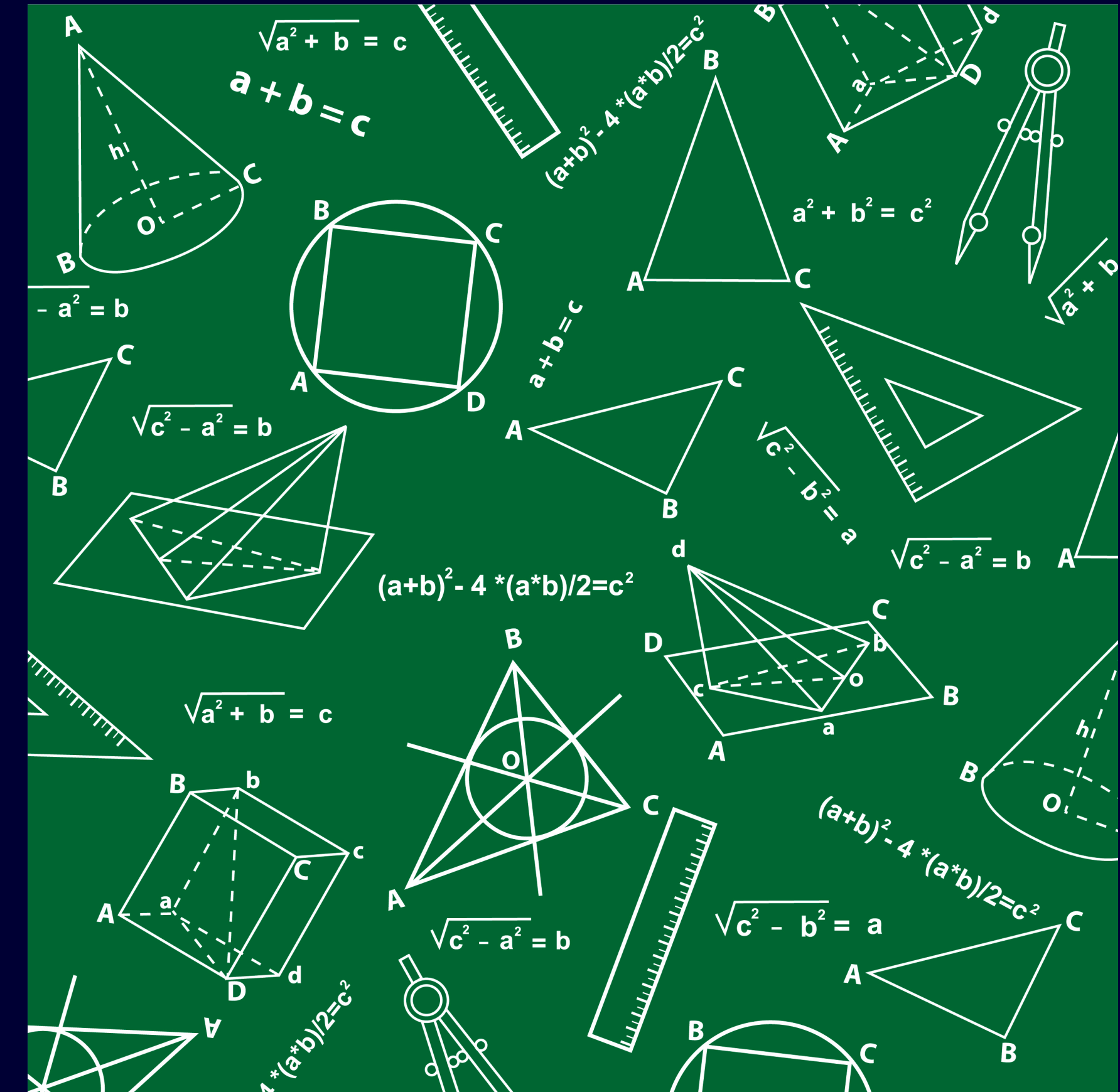
Alex Kontorovich

What does Trigonometry look like in school?

Sine	Cosine	Tangent
$\frac{\text{opposite}}{\text{hypotenuse}}$	$\frac{\text{adjacent}}{\text{hypotenuse}}$	$\frac{\text{opposite}}{\text{adjacent}}$
SOH	CAH	TOA

www.mathwarehouse.com

$\tan \theta = \frac{O}{A}$
 $\sin \theta = \frac{O}{H}$
 $\cos \theta = \frac{A}{H}$



SOHCAHTOA

SOH $\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$
CAH $\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$
TOA $\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$

But how did we get here? And why?

We no longer look UP!

But before artificial light (300,000 - 150 years), EVERY night looked like ...

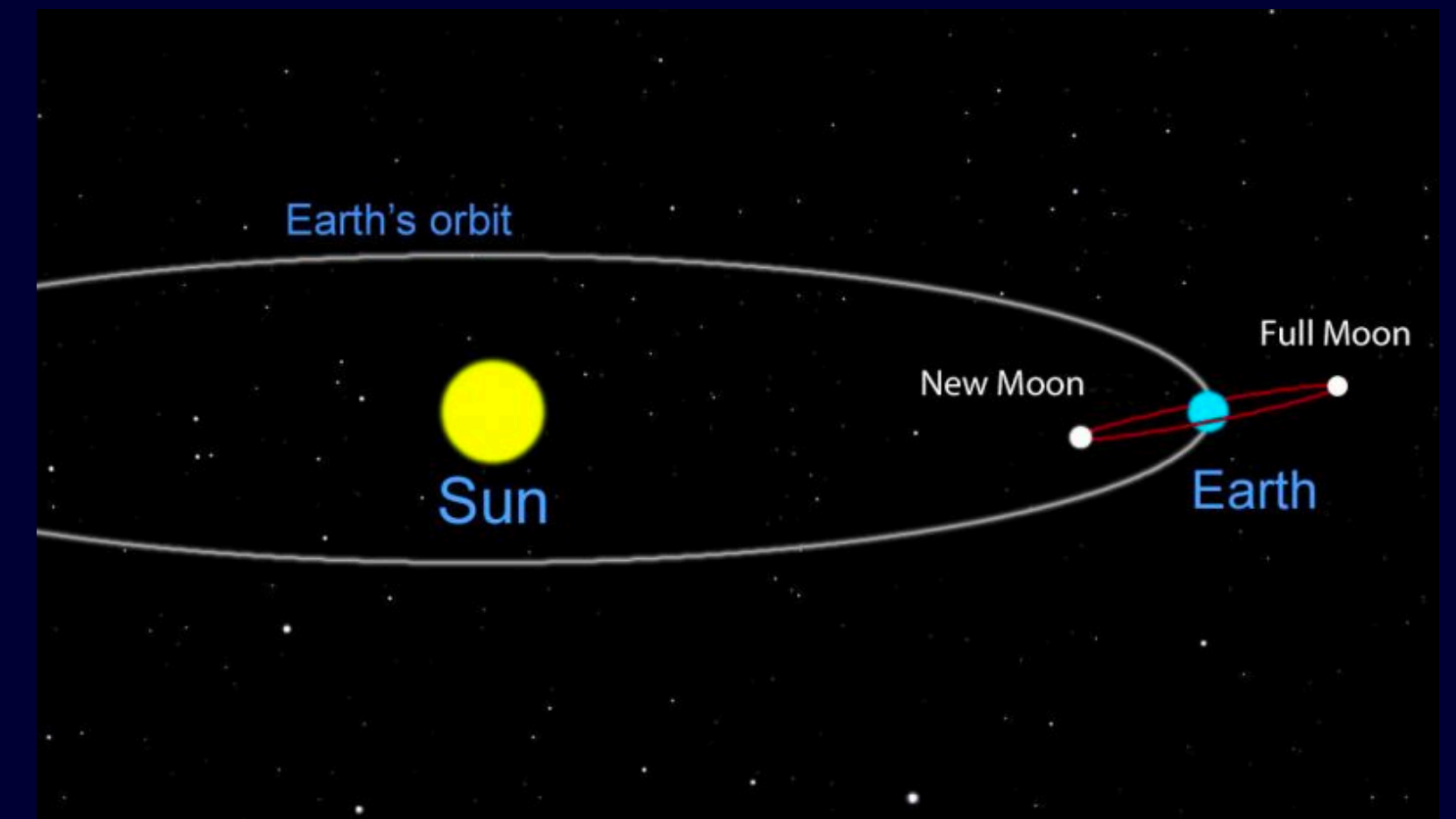


Polaris



What could you figure out on your own?

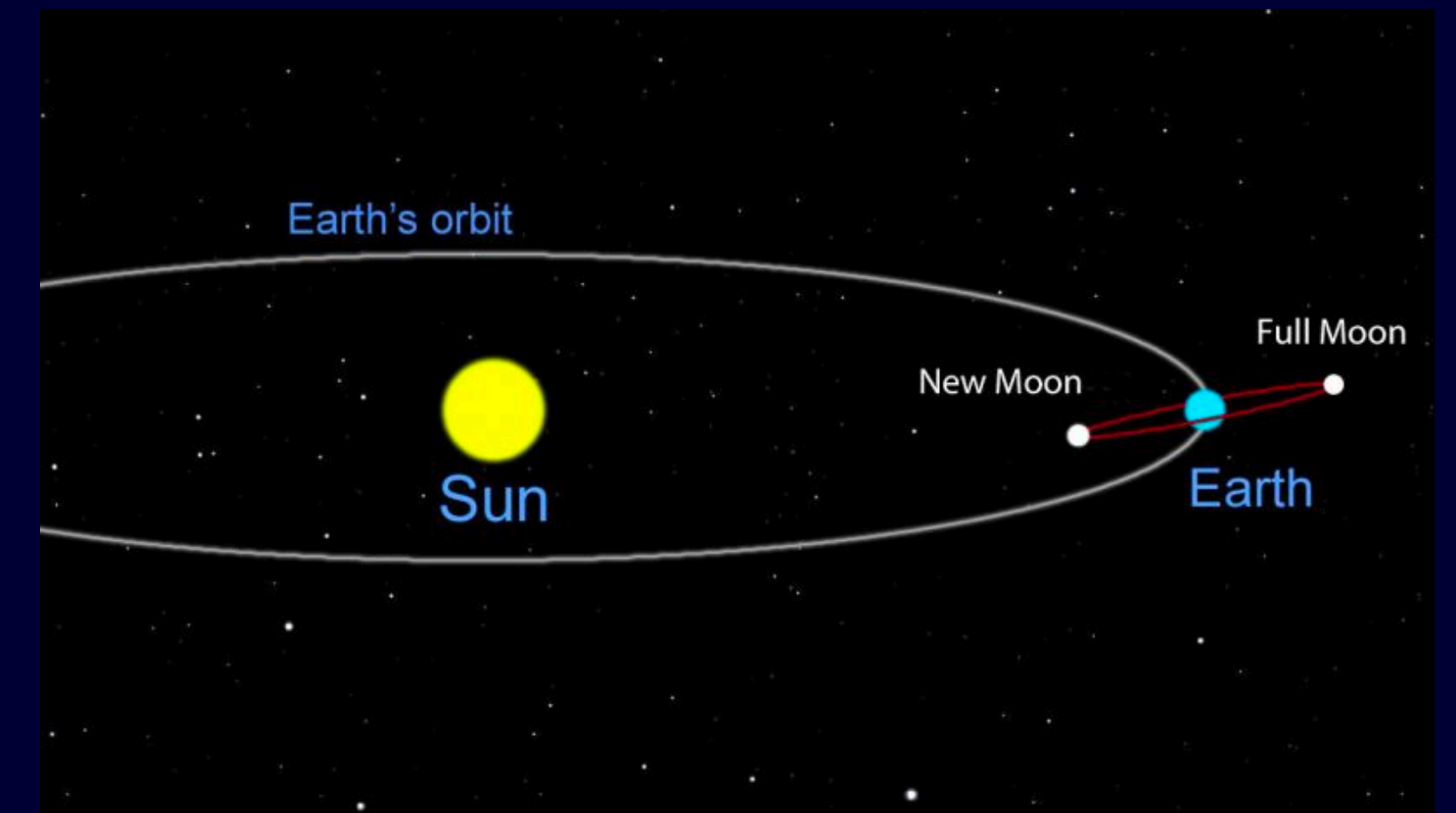
***Every* advanced civilization (Babylonians, Egyptians, Chinese, Mayans, Indians, Greeks, etc etc) discovered that a “year” is about 365 days, and a “moon”th is about 28 days, and moon phases (new, half, full, half) taking 7 days.**



What could you figure out on your own?

How to keep track of time? Calendars could track:

- Just position of the Sun
- Just position of the Moon
- Both Moon and Sun

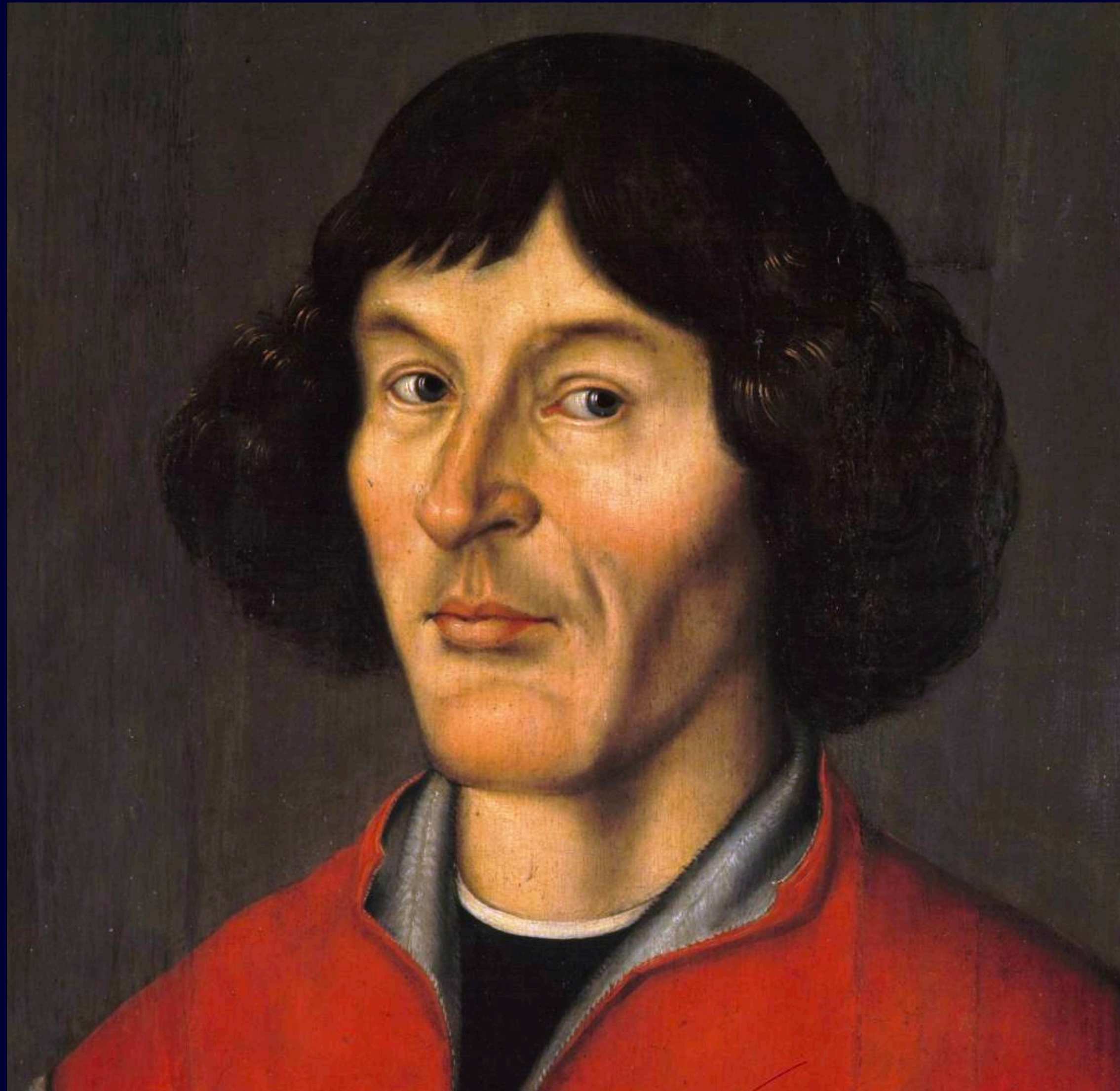


Curiously, 3.8 Billion people (half of the world)
Follow Abrahamic religions (Judaism, Christianity, Islam),
whose three calendars each made a different choice!

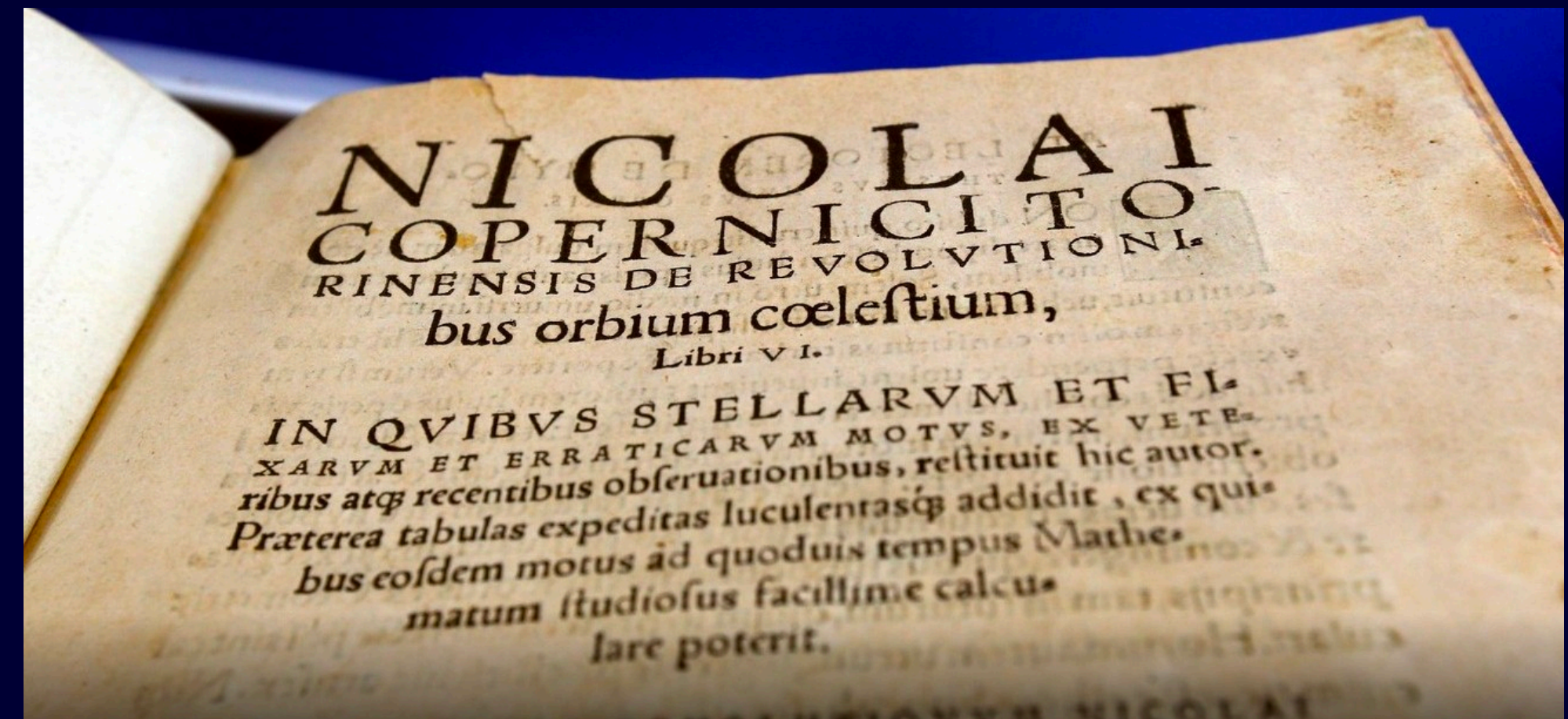
Modern reinterpretation:



Many people learn that Nicolaus Copernicus (~1500)



first suggested “helio-”
(as opposed to “geo-”)
centrism.



We know about Galileo,
Kepler, Brahe, Newton,
Hubble, etc etc, right?

Today we know that:



Circumference of Earth $\approx 25,000$ mi = πD

Diameter of the Earth $\approx 8,000$ mi

Diameter of the Moon $\approx 2,000$ mi (1/4 of Earth's)

Diameter of the Sun $\approx 800,000$ mi (100x Earth)

Distance to the Moon $\approx 250,000$ mi

Distance to the Sun $\approx 100,000,000$ mi (400x Moon's)

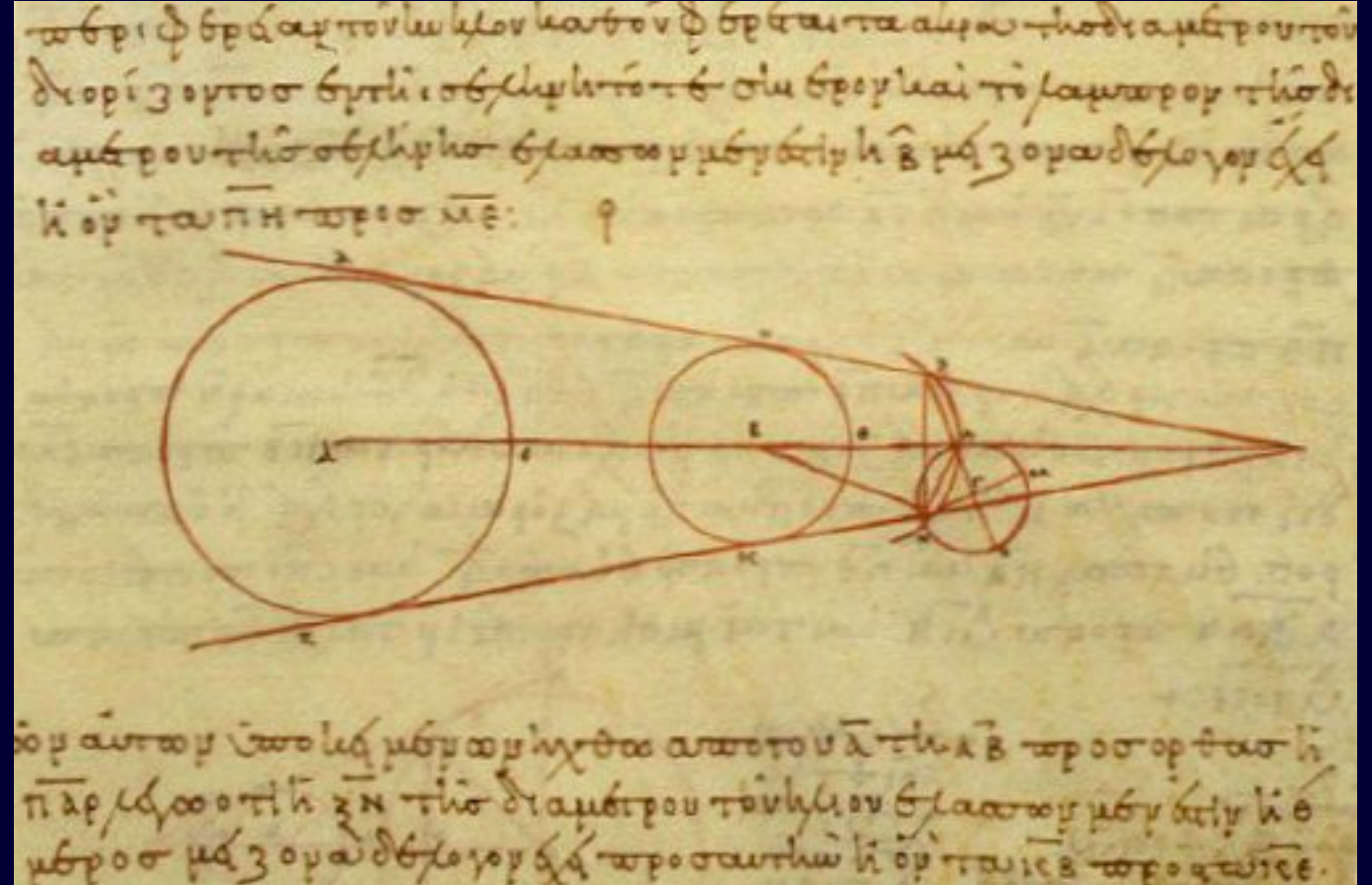
You couldn't possibly figure any of this out without modern, fancy equipment, right???

Aristarchus of Samos (310-230 BC)



Showcased the Power of Trigonometry:
Measuring That Which You Cannot Touch!

Using only his mind, together with
extremely basic observations, (almost)
computed all of these Earth/Moon/
Sun diameters and distances!



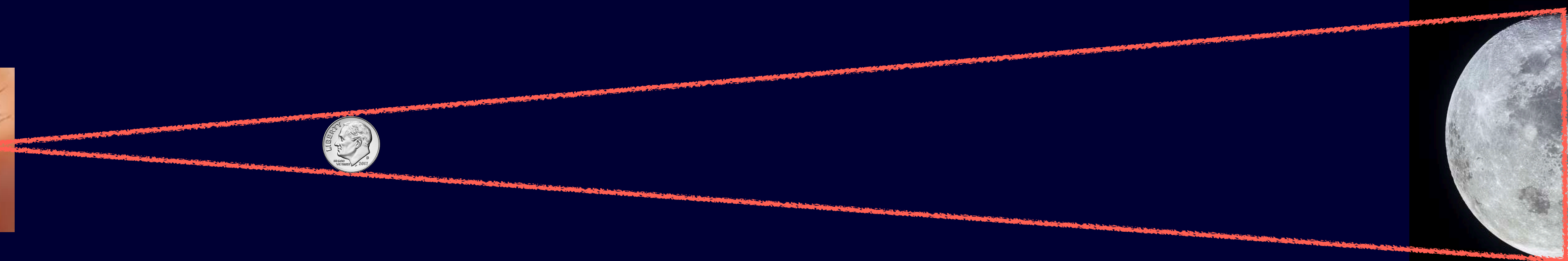
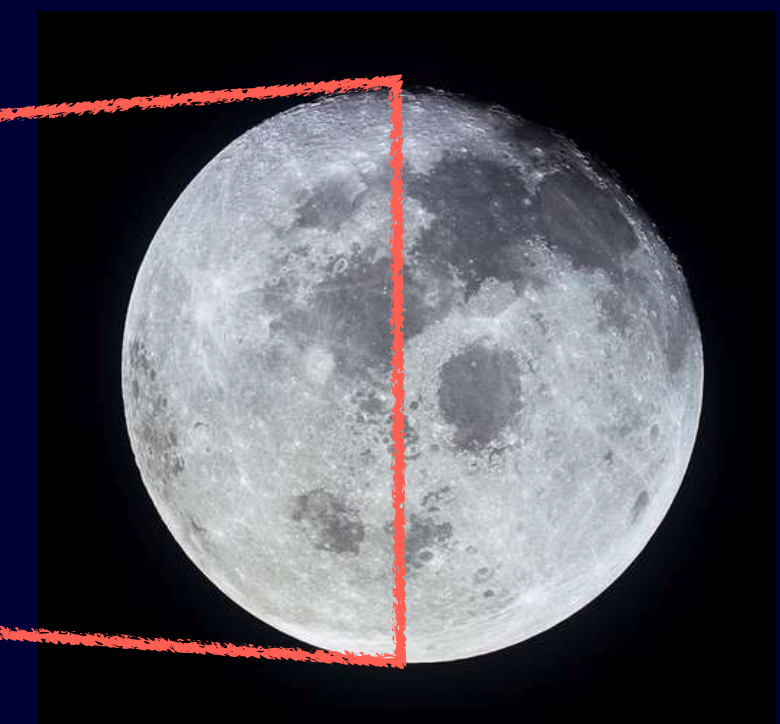
Let's compute ALL of these in 4 "easy" steps:

Step 1: Compute diameter/distance for the Moon:

Take a dime (0.7" diameter). Look at a full Moon.



Move dime away until Moon is perfectly covered. (90")

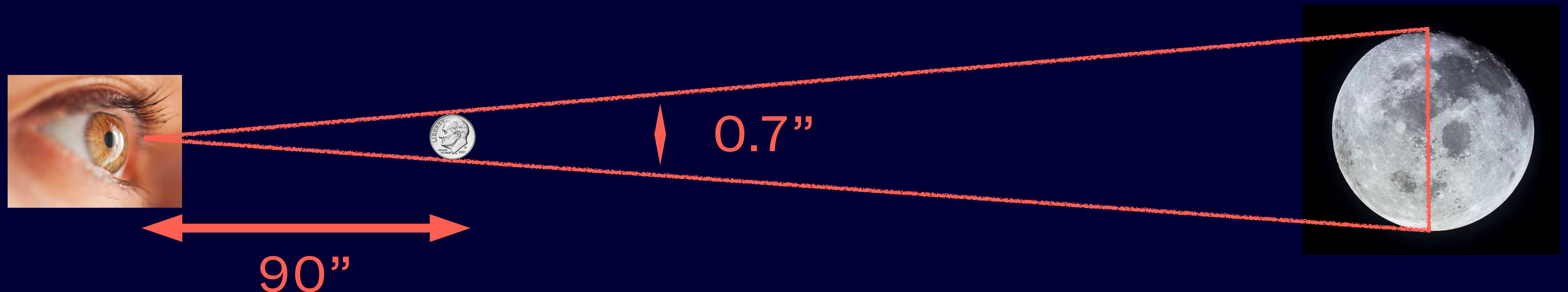


Let's compute ALL of these in 4 "easy" steps:

Step 1: Compute diameter/distance for the Moon:

Take a dime (0.7" diameter). Look at a full Moon.

Move dime away until Moon is perfectly covered. (90")



By similar triangles (basic trigonometry!),

$$0.7''/90'' \approx 0.008 = \text{Diameter(Moon)}/\text{distance(Moon)}$$

Because of total solar eclipses, same for the Sun!



Step 2: Aristarchus: “On the Half-Moon and Sun”



Step 2: Aristarchus: "On the Half-Moon and Sun"



Step 2: Aristarchus: "On the Half-Moon and Sun"



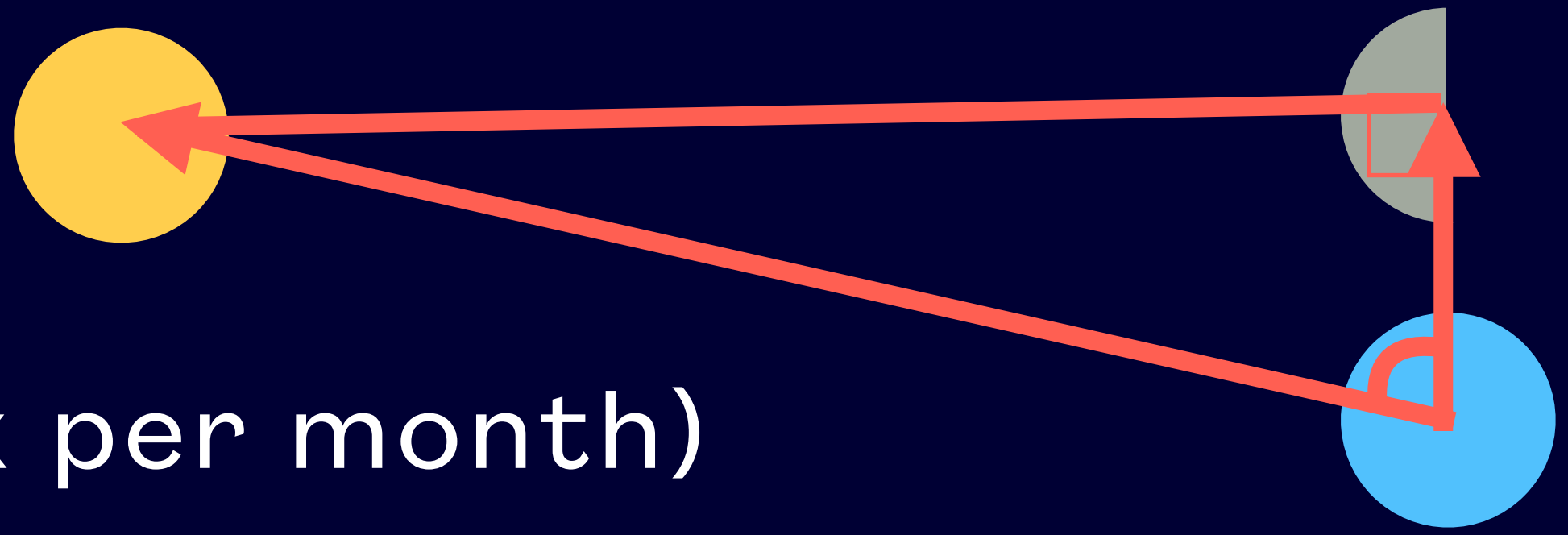
It's "always" a half-moon. But we only see it as such when E-M-S angle is 90° !

When you measure, M-E-S angle is also nearly 90° !

So the Sun is VERY far away!

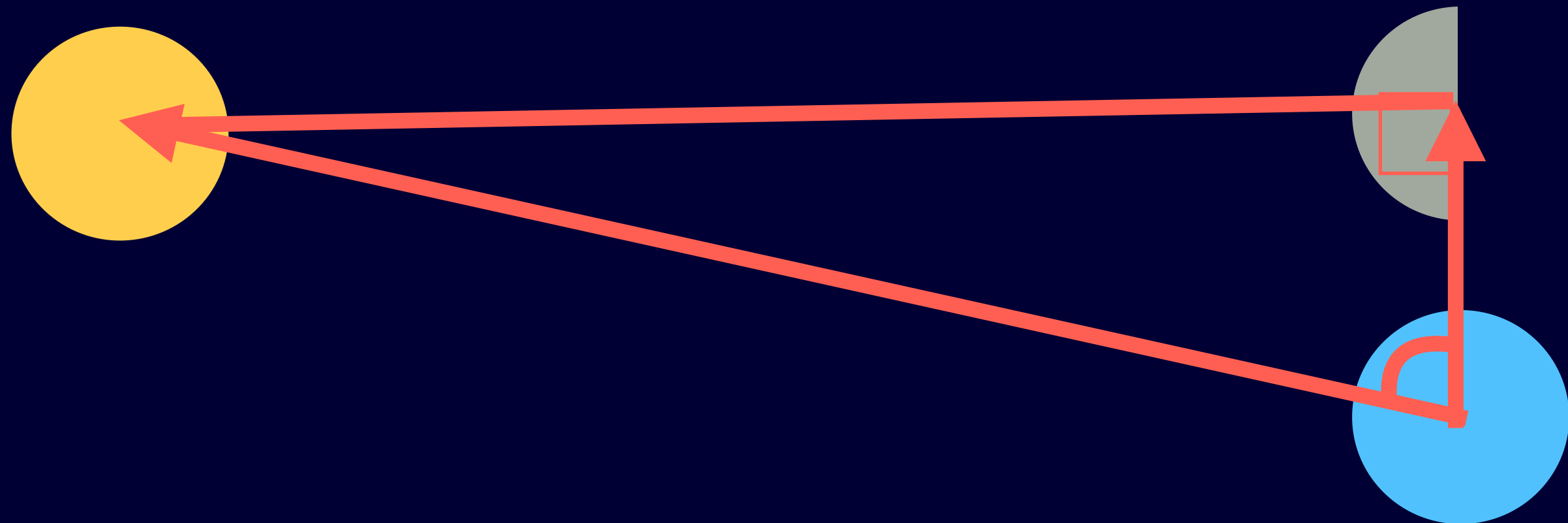
Which means (eclipses) the Sun is HUGE!!!

(At least as compared to the Moon)



(2x per month)

Step 2: Aristarchus: "On the Half-Moon and Sun"



When you measure, M-E-S angle is also nearly 90° !

So the Sun is VERY far away!

Which means (eclipses) the Sun is HUGE!!!

Remember:

(At least as compared to the Moon)

$$\text{Diam}(\text{Moon})/\text{dist}(\text{Moon}) = 0.008 = 2/250$$

Same with Sun (please don't look directly at it!)

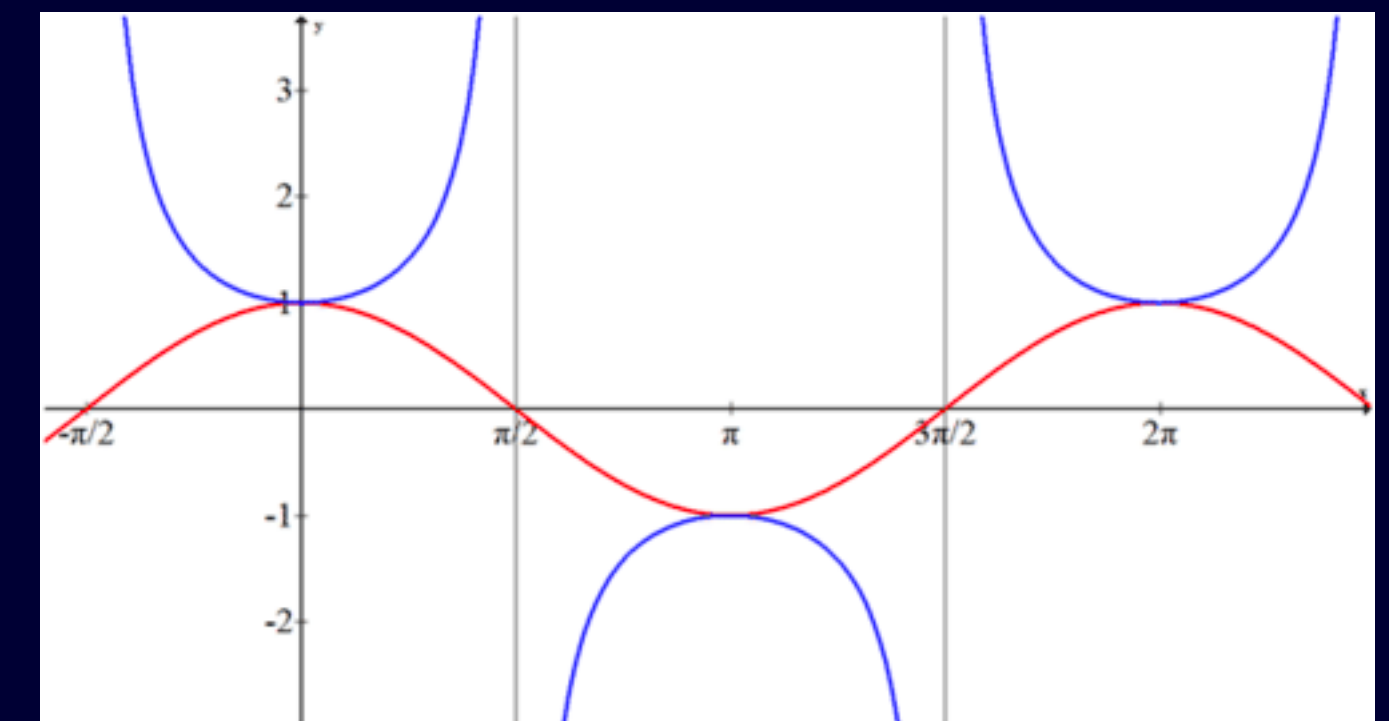
$$\text{Diam}(\text{Sun})/\text{dist}(\text{Sun}) = 0.008 = 800/100,000$$

The mystery angle measures: $\approx 89.86^\circ$.

$$\text{Trig: } \text{dist}(\text{Moon})/\text{dist}(\text{Sun}) = \text{Cos}(89.86^\circ)$$

Very unstable/error-prone!!!

$$\text{Trig: } \text{dist}(\text{Sun}) = \text{Sec}(89.86^\circ) \times \text{dist}(\text{Moon}) \approx 400 \text{ dist}(\text{Moon})$$



Recap after Step 1 and Step 2:

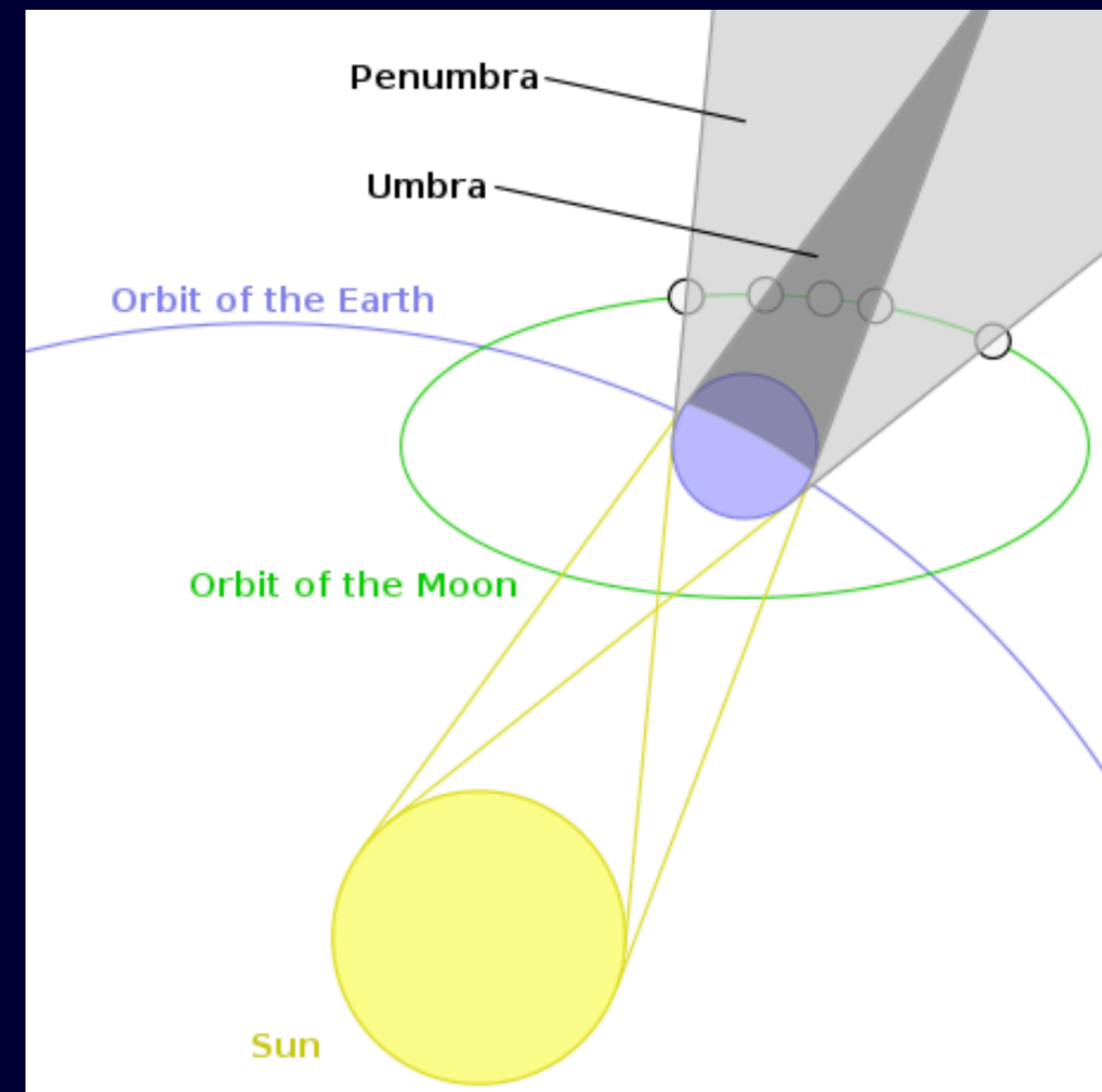
So far, we know that:

- By dime/eyes: $\text{Diam}(\text{Moon})/\text{dist}(\text{Moon}) = 0.008$
- By solar eclipses: $\text{Diam}(\text{Sun})/\text{dist}(\text{Sun}) = 0.008$
- Now also: $\text{dist}(\text{Sun}) = 400 \text{ dist}(\text{Moon})$
- And hence similarly: $\text{Diam}(\text{Sun}) = 400 \text{ Diam}(\text{Moon})$

But we still don't have enough information to solve for all the unknowns...

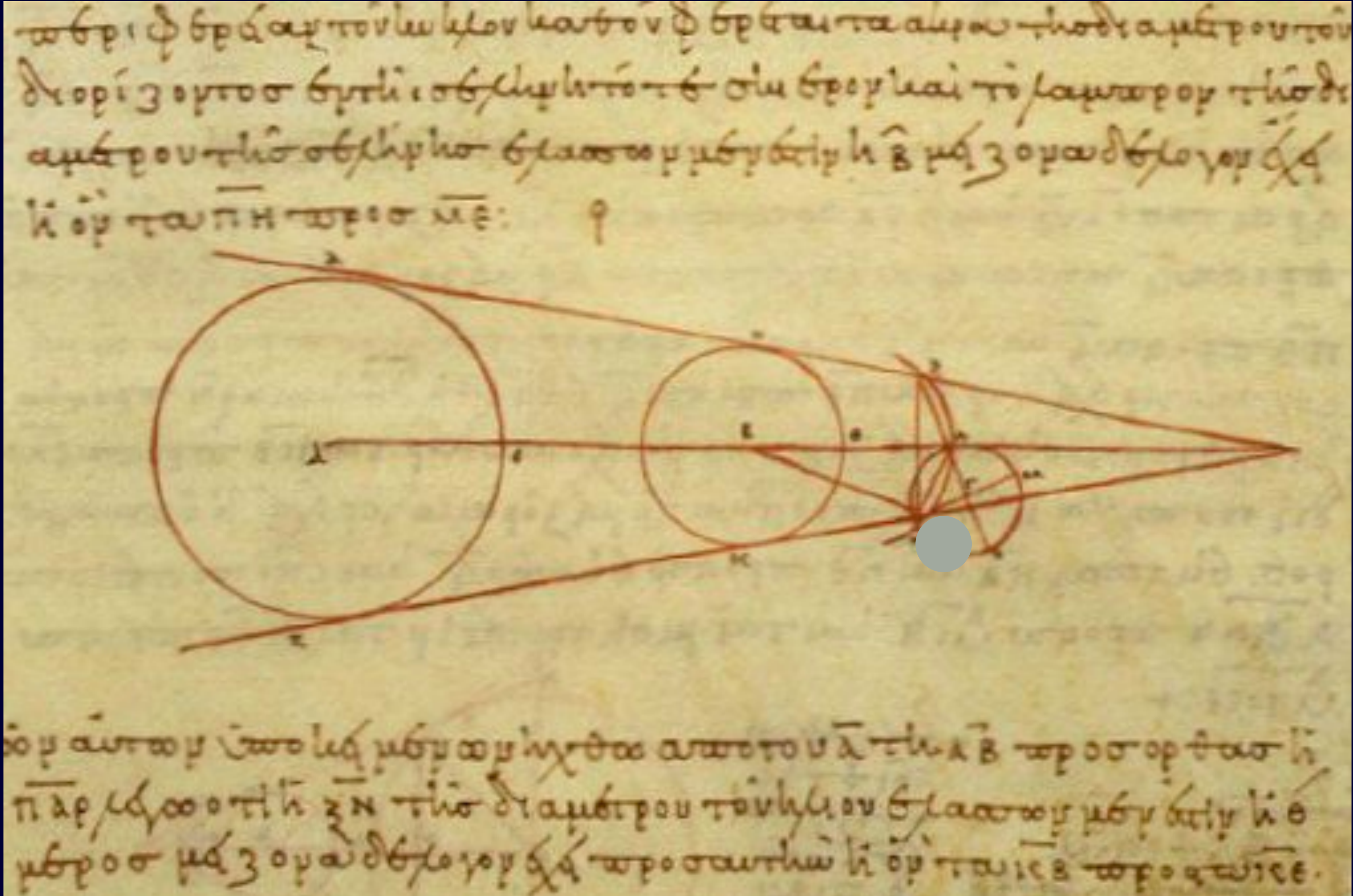
Step 3: (Still due to Aristarchus!) Connect to Earth measure

Next brilliant insight requires a **lunar** eclipse (~2x/yr)



Step 3: (Still due to Aristarchus!) Connect to Earth measure

Next brilliant insight requires a lunar eclipse (~2x/yr)

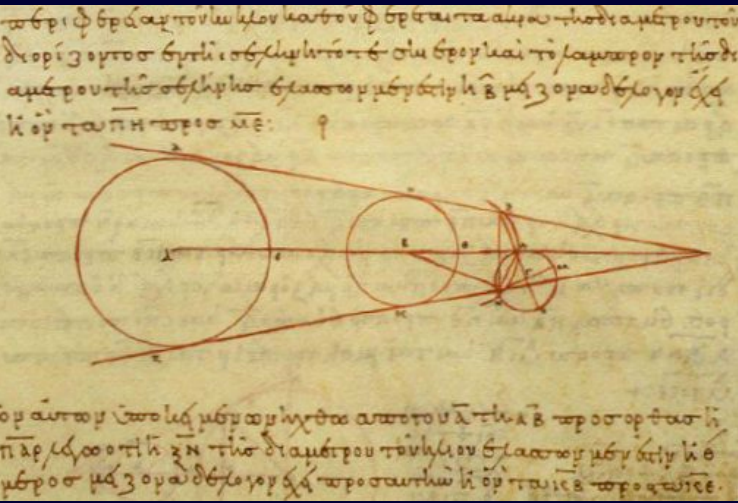


Measure time from entering totality to exiting: 3 hrs.
Also measure how long it takes the Moon to go red: about 1 hour. And thus:

Diameter of Umbra is **3x**
Moon's Diameter!

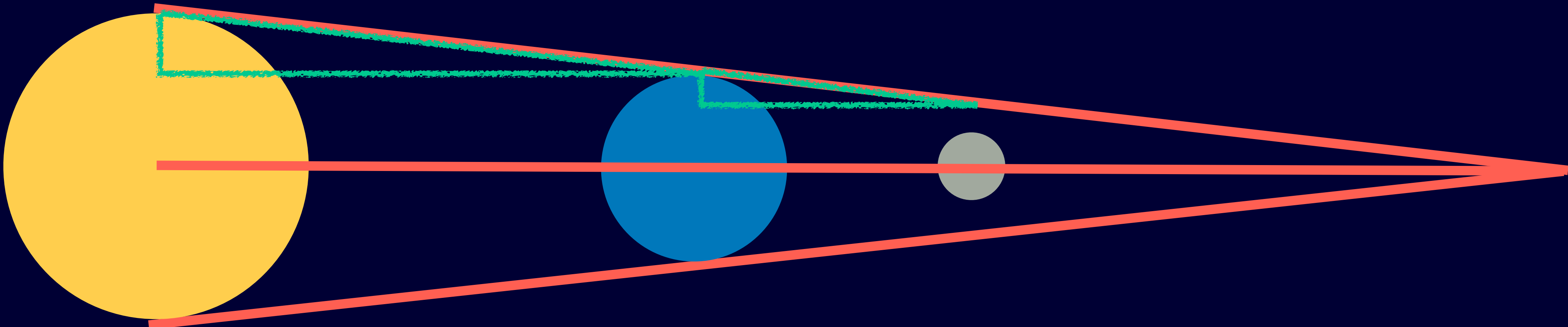
Step 3: (Still due to Aristarchus!) Connect to Earth measure

Next brilliant insight requires a lunar eclipse (~2x/yr)



Diameter of Umbra is **3x** Moon's Diameter

$$\frac{dist(Sun)}{dist(Moon)} = \frac{Rad(Sun) - Rad(Earth)}{Rad(Earth) - Rad(Umbra)}$$

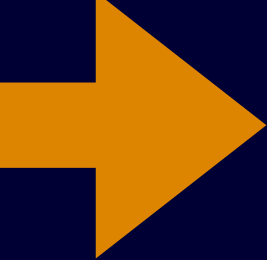


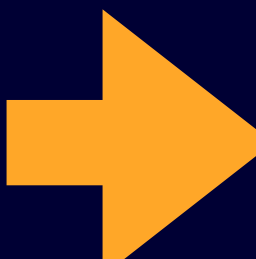
Step 3: (Still due to Aristarchus!) Connect to Earth measure

Next brilliant insight requires a lunar eclipse (~2x/yr)

Diameter of Umbra is **3x** Moon's Diameter

$$\frac{Rad(Sun)}{Rad(Moon)} = \frac{dist(Sun)}{dist(Moon)} = \frac{Rad(Sun) - Rad(Earth)}{Rad(Earth) - Rad(Umbra)}$$

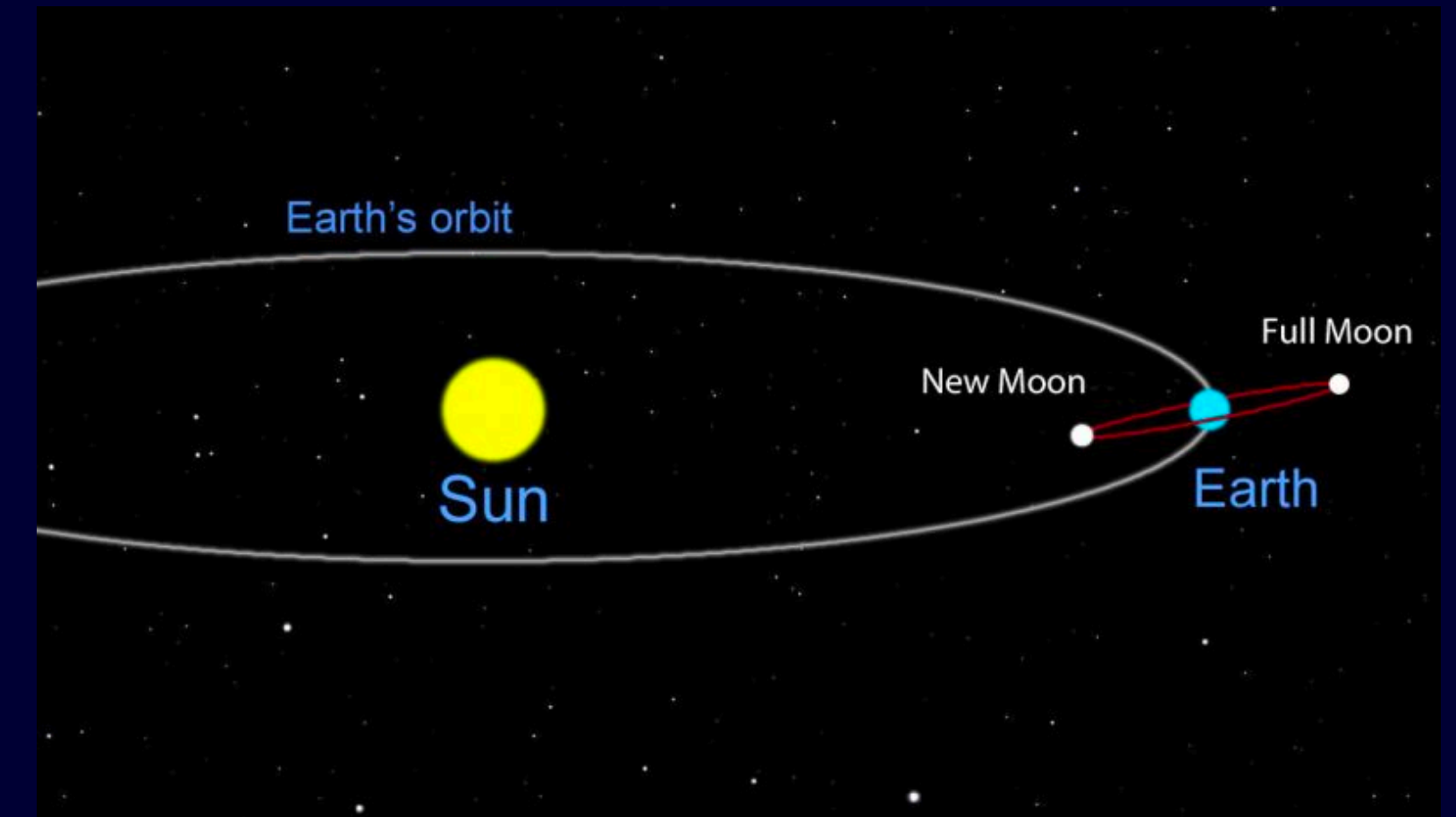

$$\frac{Rad(Sun) - Rad(Earth)}{Rad(Sun)} = \frac{Rad(Earth) - Rad(Umbra)}{Rad(Moon)}$$


$$1 - \frac{Rad(Earth)}{400Rad(Moon)} = \frac{Rad(Earth)}{Rad(Moon)} - \frac{Rad(Umbra)}{Rad(Moon)} = \frac{Rad(Earth)}{Rad(Moon)} - 3$$

$$\frac{Rad(Earth)}{Rad(Moon)} \approx 4$$

Recap after Steps 1, 2 and 3:

- $\text{Diam}(\text{Moon})/\text{dist}(\text{Moon}) = 0.008$
- $\text{Diam}(\text{Sun})/\text{dist}(\text{Sun}) = 0.008$
- $\text{dist}(\text{Sun}) = 400 \text{ dist}(\text{Moon})$
- $\text{Diam}(\text{Sun}) = 400 \text{ Diam}(\text{Moon})$
- $\text{Diam}(\text{Earth}) = 4 \text{ Diam}(\text{Moon})$
- Thus $\text{Diam}(\text{Sun}) = 100 \text{ Diam}(\text{Earth}) !!!$



So maybe we revolve around the Sun??? (Heliocentrism!)

This is “all” Aristarchus could show. He still couldn't determine any *one* of these values!

Contemporaries: If Earth was moving, we would see parallax in celestial sphere, but we don't (need telescopes!), so you're wrong.

Eratosthenes (276 – 194 BC)

(Contemporary and friend of Archimedes (287 – 212 BC))

Poet, Music Theorist, Geographer, etc

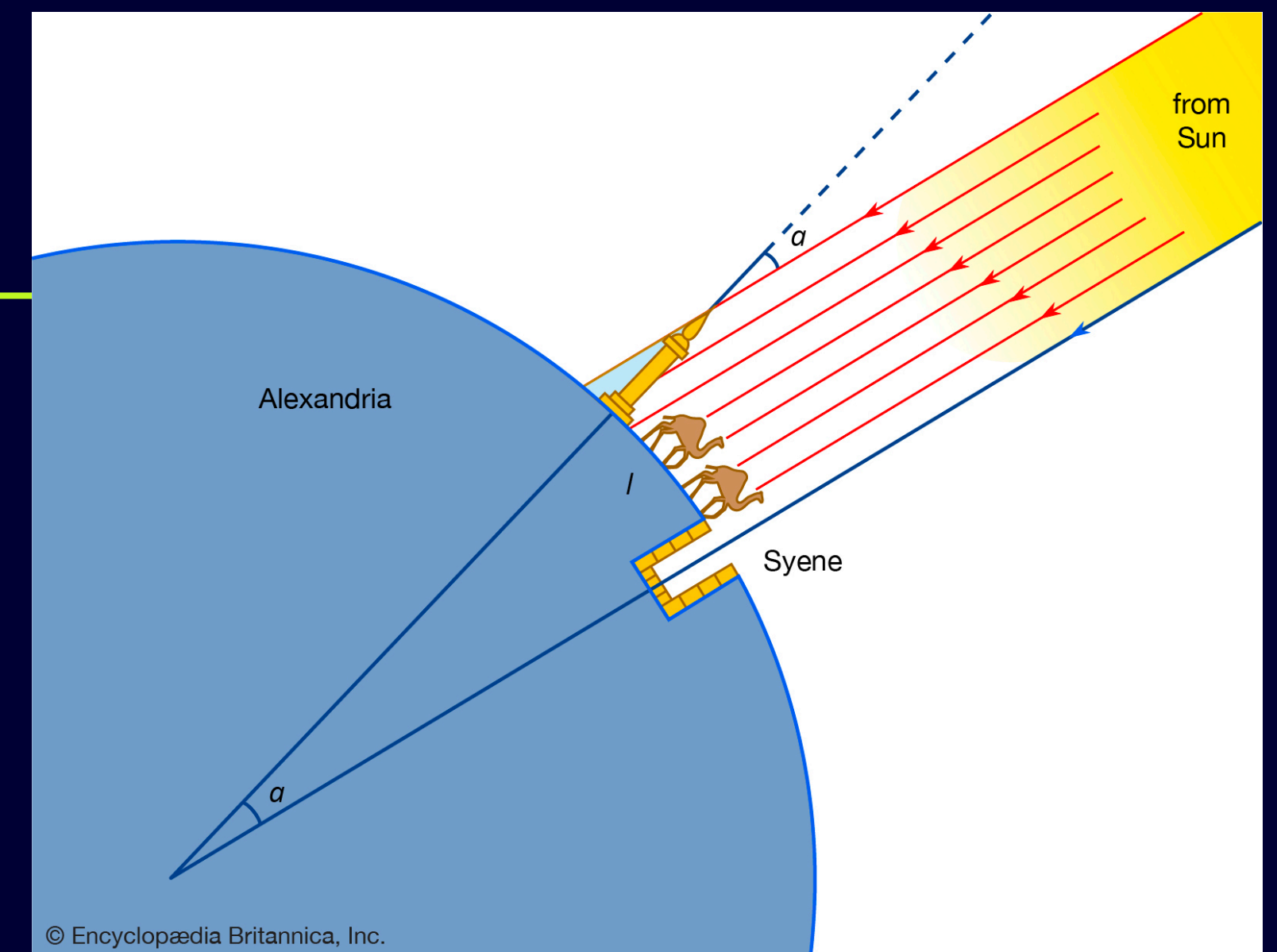
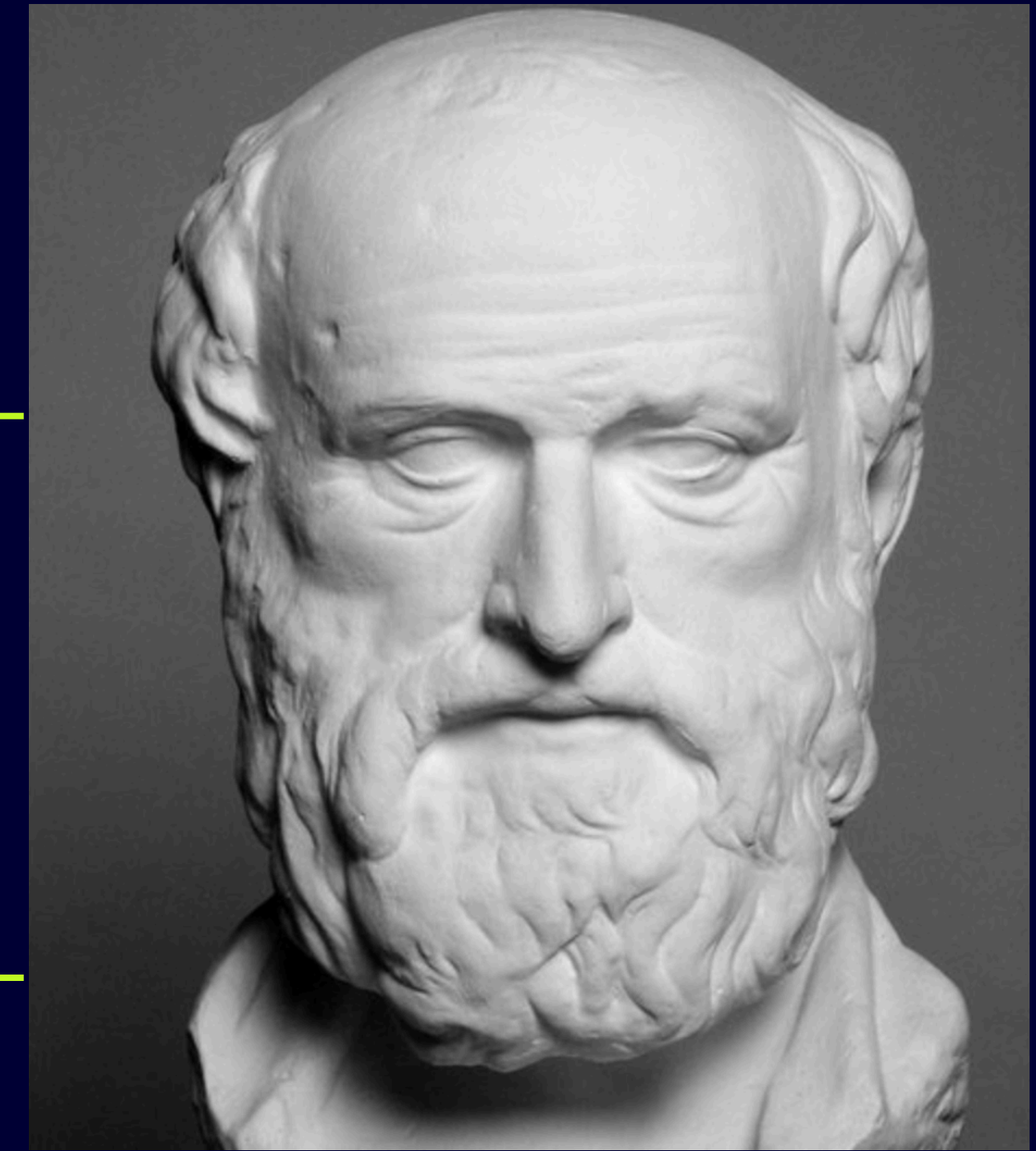
Chief Librarian at Alexandria

Introduced “Sieve of Eratosthenes” in

Number Theory

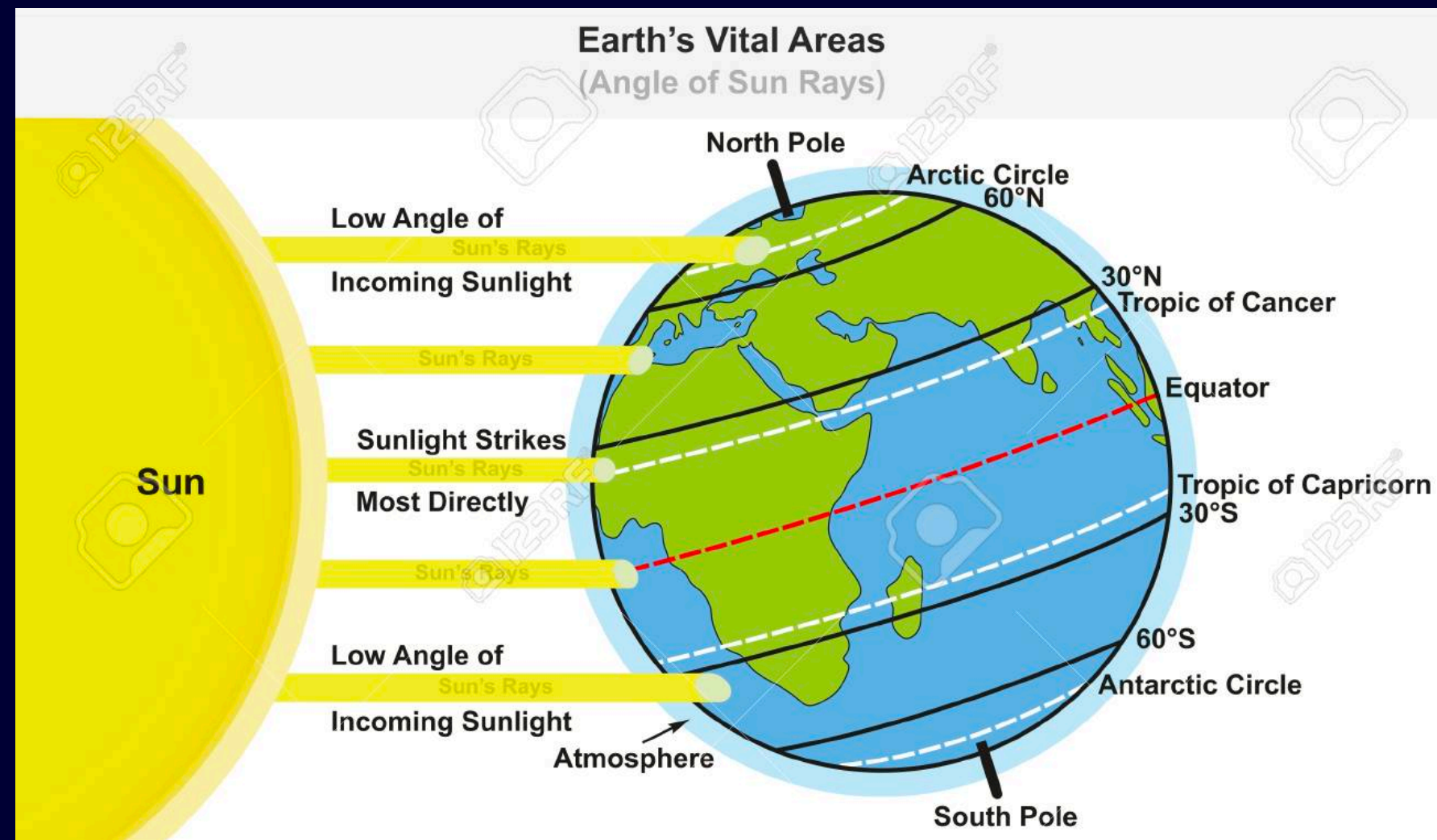
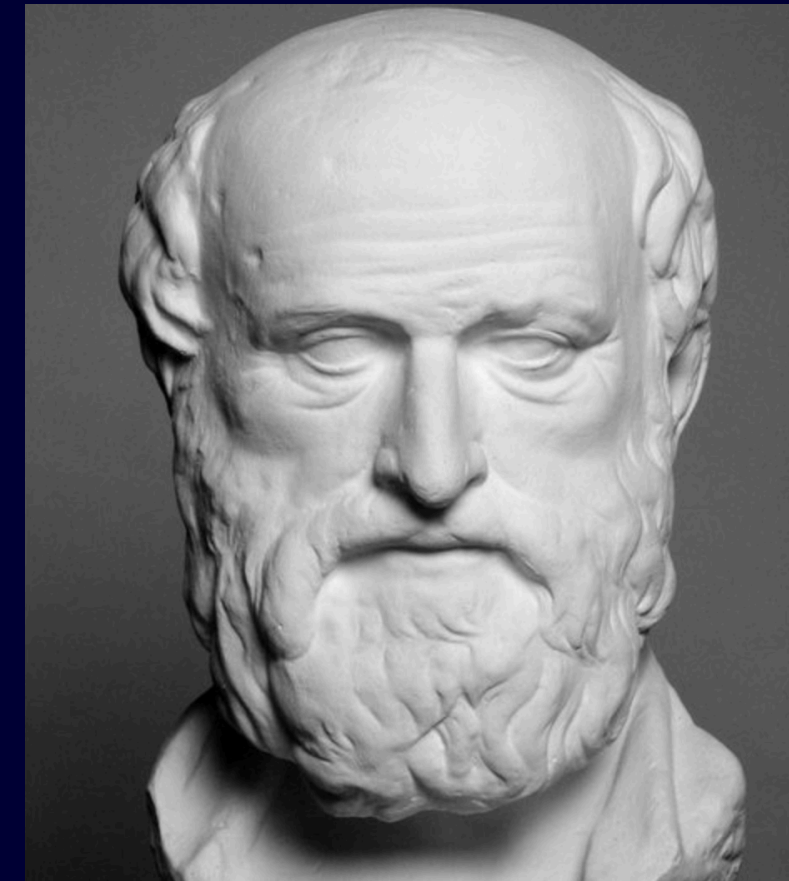
Provided Last Ingredient for us:

Circumference of the Earth!

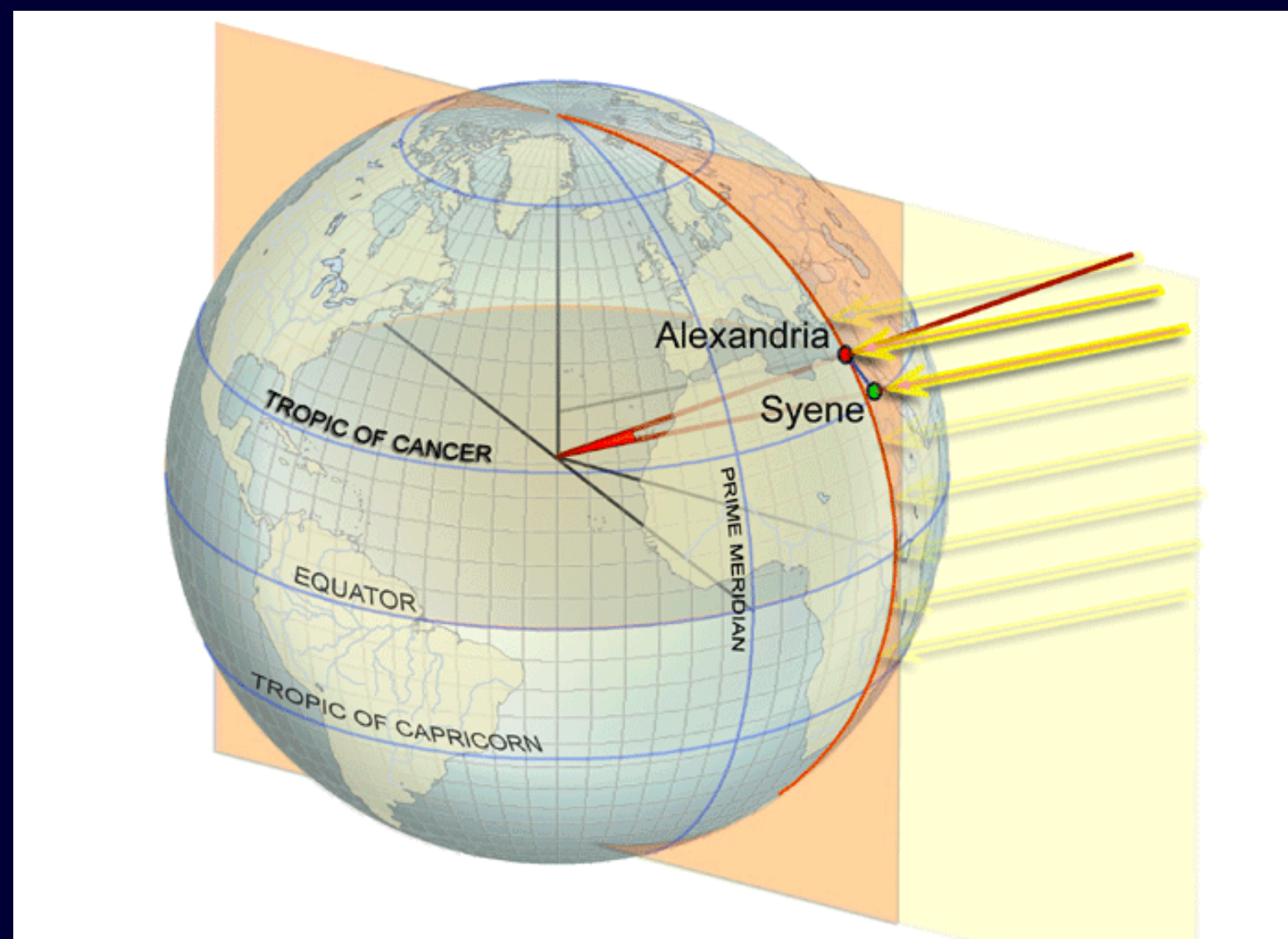


Eratosthenes (~250 BC)

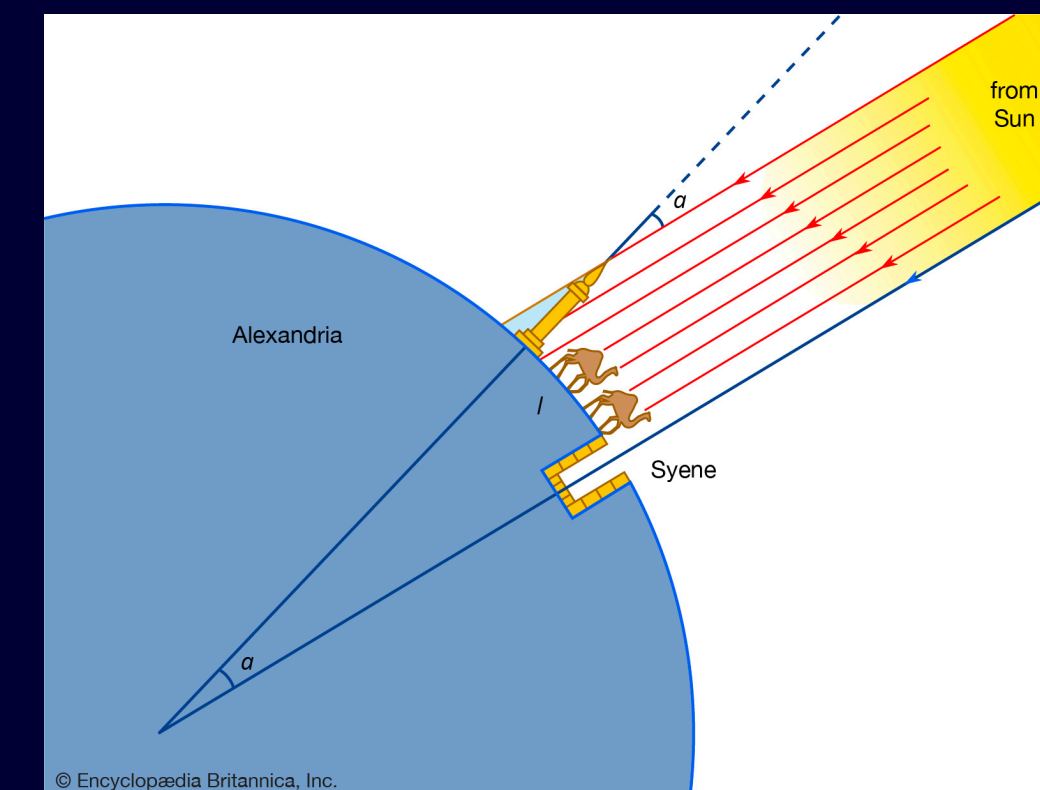
Knew:



- Earth's axis (to Polaris) is on an angle $\approx 23^\circ$ relative to ecliptic (plane around Sun)
- Syene (modern Aswan, Egypt) is \approx due South from Alexandria, ≈ 500 mi. (50 days walk on camels ~ 10 mi/day.)



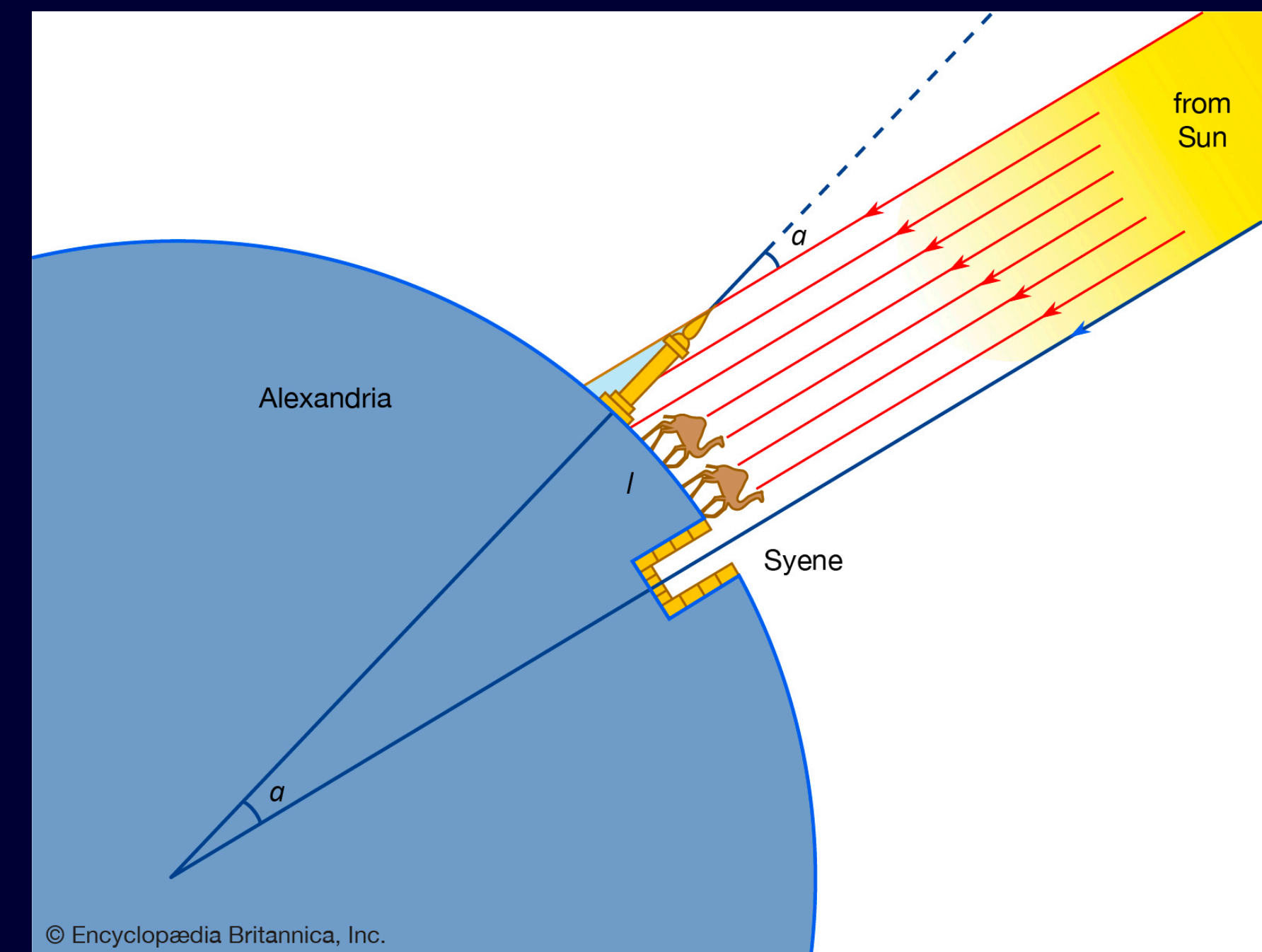
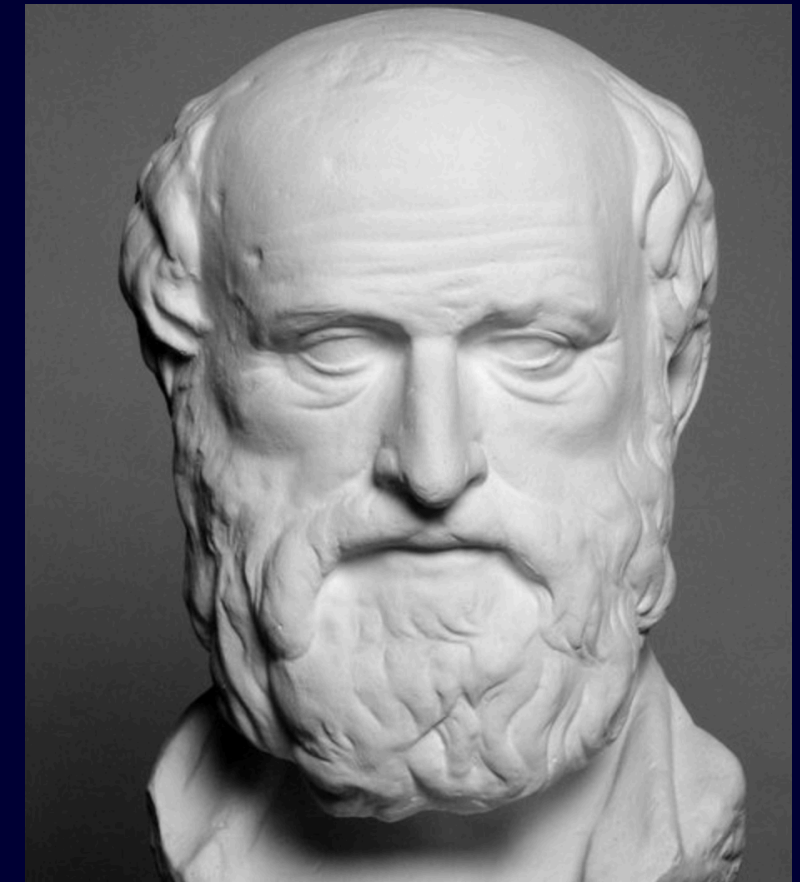
- High noon on summer solstice, bottom of well in Syene lights up!
- Same time in Alexandria, obelisk casts a shadow...



Eratosthenes (~250 BC)

- Syene (modern Aswan, Egypt) is \approx due South from Alexandria, \approx 500 mi.
- High noon on summer solstice, bottom of well in Syene lights up!
- Same time in Alexandria, obelisk casts a shadow: \approx 1/50th of circle (7°)

➔ Full Circumference $\approx 50 \times 500$ mi!!!
(Step 4!)



And that's IT! The "heavens" have been tamed by Trigonometry!

Circumference of Earth $\approx 25,000$ mi = πD

Diameter of the Earth $\approx 8,000$ mi

Diameter of the Moon $\approx 2,000$ mi (1/4 of Earth's)

Diameter of the Sun $\approx 800,000$ mi (100x Earth)

Distance to the Moon $\approx 250,000$ mi (Diam(Moon)/0.008)

Distance to the Sun $\approx 100,000,000$ mi (400x Moon's)

And all without a space ship, or even a telescope,
just the power of human imagination, ingenuity!