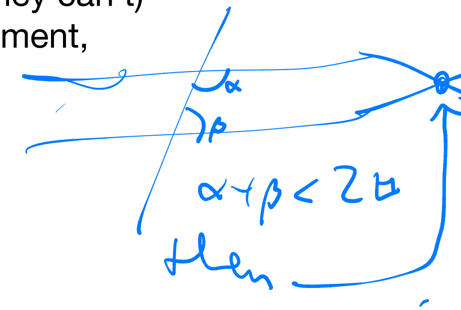
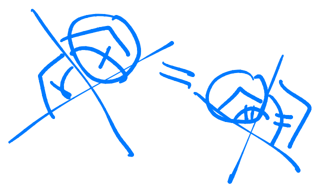
Topics for the Quiz:
Book I
Definitions (that don't really define, since they can't)
Postulates, Fifth postulate. Know the statement,

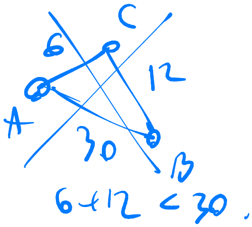


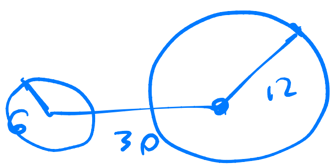
Equivalent axioms:
- sum of angles in a triangle = 180
- given a line and point off the line, there is a unique parallel thru point.

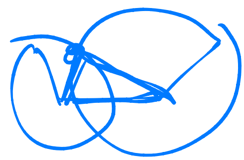
(P4: all right angles are =. Why is this necessary?... The definition of right angle is that when two lines meet, the adjacent angles are =. If two other lines also meet and have adjacent angles =, how can I compare the equality of angles that aren't adjancent)



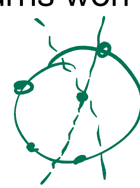
I.1: equilateral triangle, regular 3-gon. Fails to be completely rigorous, he's missing axioms that guarantee intersections between objects.





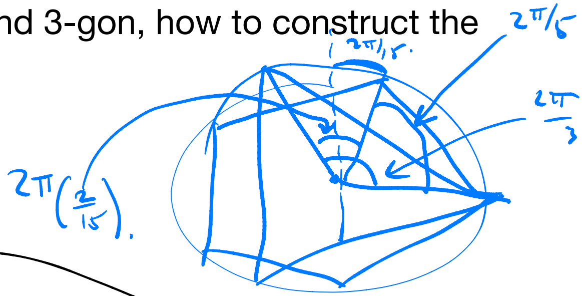


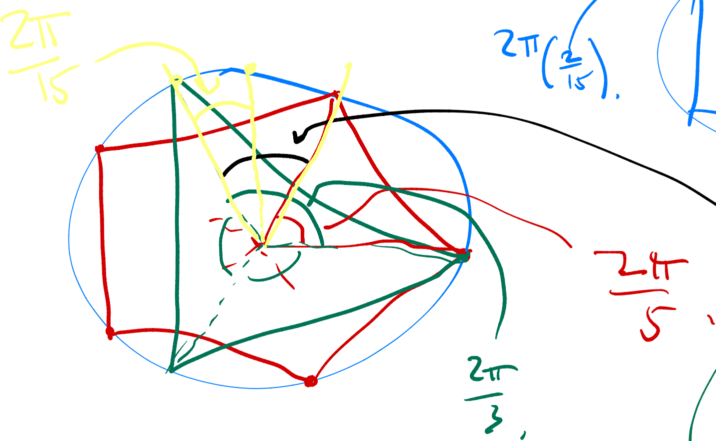
I.48 : Converse to Pyth theorem
I.47: Pyth theorem, windmill proof (Big theorem)
I.46: construct square, why not 3 right turns, why use parallel lines (it's not true in other geometries - making 3 right-angled turns won't , so there can't be a proof that avoids parallel lines...)



Book II : ends with Quadrature of Rectangle
Book III: circles/tangents, how to find the center of a circle
Book IV: regular n-gons, constructs 5-gon, (+3-gon) constructs

the 15-gon. Given the 5-gon and 3-gon, how to construct the 15-gon.





$$\boxed{\frac{2\pi}{3} - \frac{2\pi}{5} = 2\pi\left(\frac{2}{15}\right)}$$

" green-red.

$$= 2\pi\left(\frac{1}{3} - \frac{1}{5}\right)$$

$$= 2\pi\left(\frac{5}{15} - \frac{3}{15}\right)$$

Noneuclidean geometry: Lobachevsky, Bolyai, Gauss (1830s), Riemann (no, lines don't have to be infinitely long, just unbounded - P2), Beltrami (1868), models, in particular, the disk model, MC Escher (1950s) rediscovering Poincare's model for the hyperbolic disk. What does line segment mean in Poincare disk?

"plane" = unit disk.

Q: Handdraw line through them.

Hand draw line through them

Answer: circular arc that is orthogonal (at right angle) to the bounding circle

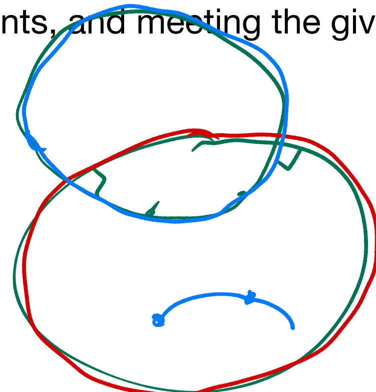
Sample Q: Why does the existence of other models prove that the Parallel Postulate cannot be proved from the other 4?
Answer: Every FACT in hyperbolic model is a FACT in Euclidean model, just with the "undefined terms" "point" "line" "plane" "circle" shuffled around. So there are no new contradictions in the hyperbolic model that didn't already exist in the Euclidean one. These two models are "equiconsistent", each is just as valid as the others.

Recall: in hyperbolic plane, parallels do exist, too many do! In spherical model, no parallel lines

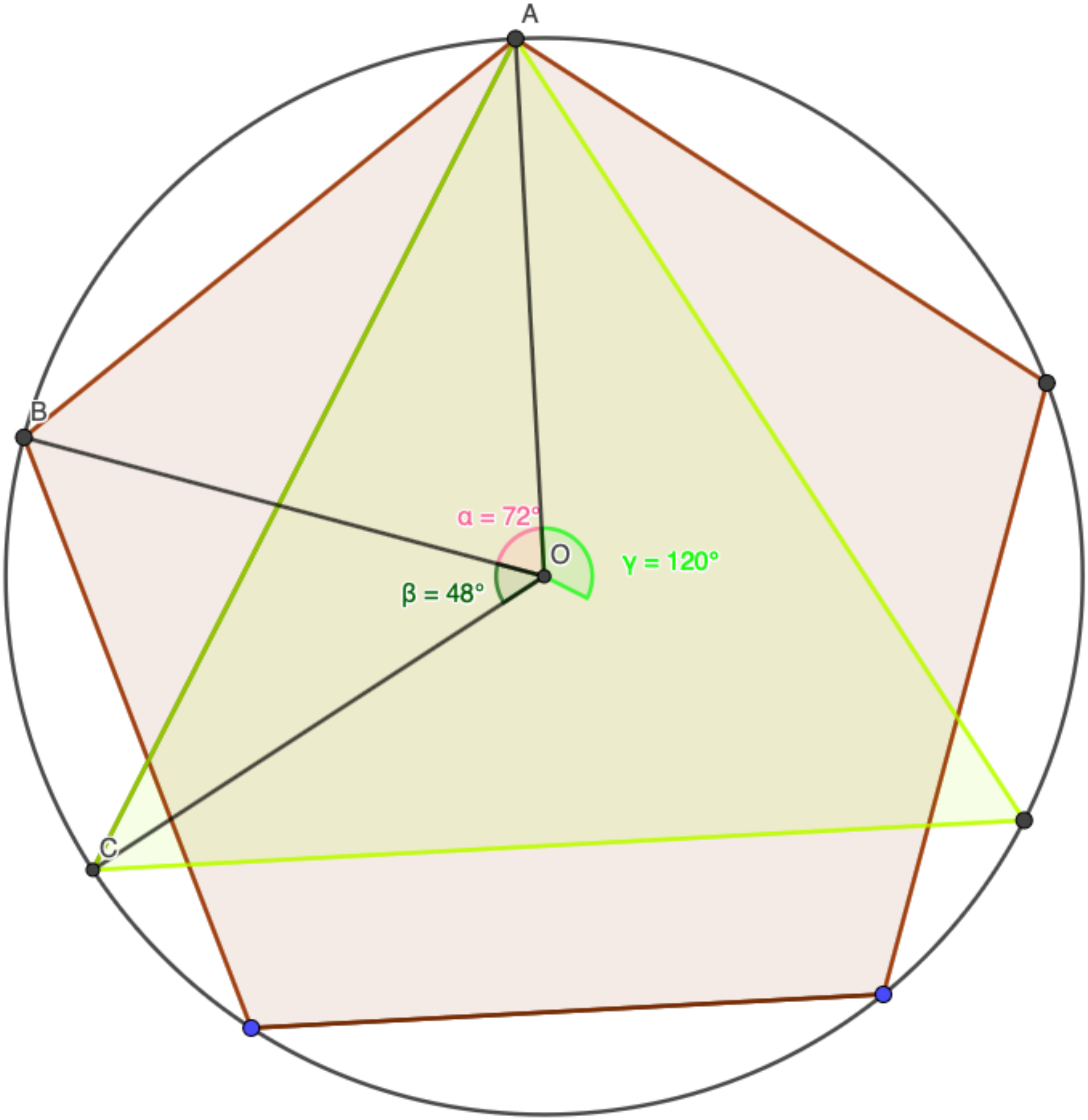
Sample Q: in the Poincare model of the hyperbolic plane, given a line and point off the line, draw two different parallel lines through the point.

Example: In hyperbolic geometry, P1: given any two points, there exists a line segment through them.

What does that statement say in Euclidean geometry?
Answer: Given a circle, and two points inside the circle, there exists a further circle passing through those two points, and meeting the given circle orthogonally



1796 Gauss discovers that not only can the 3, 5, 15 gons be constructed (and all of their doubles), but also the 17-gon can be constructed.
Reason: cos (2\pi / 17) can be written in terms of square roots only, that is, it is a constructible number.
Bigger reason: 17 is a "Fermat prime",
that is, a number of the form: 1 + 2^2^m (and so are 3 and 5)
And the only constructible n-gons are these.


A
B
C
O
α = 72°
β = 48°
γ = 120°

New: Book V:

all about proportionality.

Eudoxus of Cnidus.

~~Hippasus~~

550 BCE Pyth

400 Hippocrates Lune

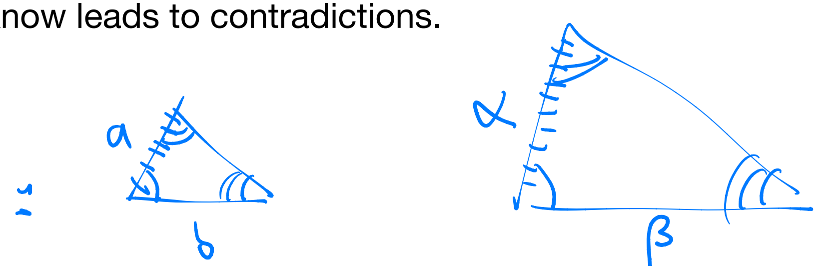
350 Alexandria

300 Euclid

The Elements

~~Axiom of Commensurability.~~

State of affairs before Eudoxus: people believe that statements that were proved using the axiom of commensurability are correct. But they know that the proofs are invalid, because they use an axiom that they know leads to contradictions.



$\cancel{\frac{7}{12} =}$ $\boxed{\frac{a}{\alpha} = \frac{b}{\beta}}$

Pythagorean approach: find a measure common to both a and alpha

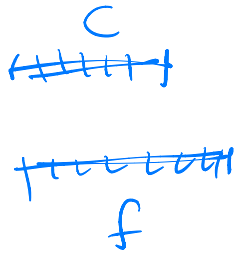
then argue that b/ beta has the same whole number ratio

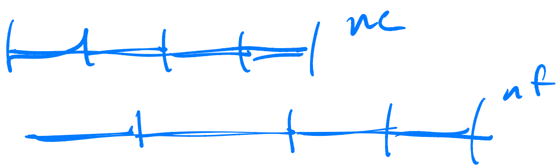
Eudoxus develops properly the theory of proportion and fixes Pythagorean geometry to not use the fallacious axiom of commensurability.

Book V is that theory.

V.15:

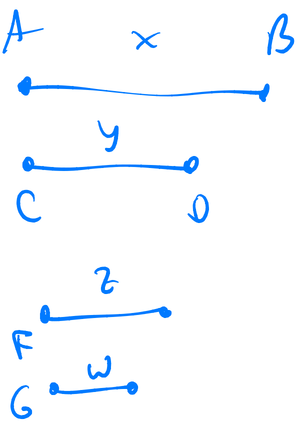
$$\boxed{\frac{c}{f} = \frac{n\cdot c}{n\cdot f}}$$





Big Boss: V.25:

Assume that $\frac{x}{y} = \frac{z}{w} = a$ with a>1.



Then $x+w > z+y$

Proof:

$x = a\cdot y$, $a > 1$.

$z = a\cdot w$

$x - z = a(y-w)$

$x - z > y - w$.