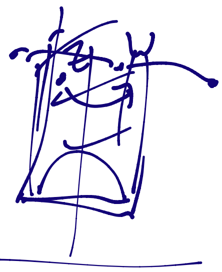
Last time: Siegel domain for $SL_2(\mathbb{Z})$

(SL_n).



Lemma: If $z, w \in \mathcal{F} = \{ |z|>1, |\mathrm{Re}\, z| < \frac{1}{2} \}$ $\boxed{w = u+iv.}$

& $\exists\, \gamma \in \Gamma$, $z = \gamma w$, then $z = w$.

Pf: Case $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $c=0$. $\begin{pmatrix} a & \\ 0 & d \end{pmatrix}$, $ad=1$,

$\gamma = \begin{pmatrix} 1 & * \\ 0 & 1 \end{pmatrix}$, $\gamma w = w + *$, $\in \mathbb{Z}$. $\Rightarrow * = 0$, $z = w$.

If not, $c \neq 0$. $\mathrm{Im}\, z > \frac{\sqrt{3}}{2}$, $\quad \frac{\sqrt{3}}{2} < \mathrm{Im}\, z = \mathrm{Im}\, \gamma w = \frac{\mathrm{Im}\, w}{|cw+d|^2} \le \frac{v}{c^2 v^2}$

$|cw+d|^2 = (cu+d)^2 + c^2 v^2 \ge c^2 v^2$

$= \frac{1}{c^2 v} < \frac{2}{c^2 \sqrt{3}}$.

$\Rightarrow c^2 < \frac{4}{3}$. $\Rightarrow c = \pm 1$.

Say $c = 1$, $\gamma = \begin{pmatrix} a & b \\ 1 & d \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ +1 & 0 \end{pmatrix}} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$

$\underbrace{\begin{pmatrix} +a & -1 \\ +1 & 0 \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}}$

$\begin{pmatrix} a & ad-1 \\ 1 & d \end{pmatrix}$