Review Sheet for Math 151 Midterm 1 Fall 2022

The following questions are intended to give you practice working problems that test the ideas covered in the course for the first midterm exam. The number of problems on this review sheet is larger than the number of problems on a typical 80-minute exam – this sheet is NOT A PRACTICE TEST. You should not memorize the problems, as the problems you encounter on the midterm exam will not look exactly like the problems on this review sheet – the exam problems will be different. Your goal is to be able to think through and understand the processes required to answer the questions correctly. Before you work the review problems, you should study for the exam and when you feel you have prepared enough, try doing the problems on this sheet WITHOUT looking at your notes, textbook, or videos. Make sure to try this a couple of days before the midterm so that you will have time to fill in any gaps of knowledge you uncover. If you start your studying by doing the review sheet first, you will not maximize the benefit of the review sheet.

1. Find the standard equation of a circle with center $(-38, 14)$ and radius 39.

$$(x+38)^2+(y-14)^2=39^2$$

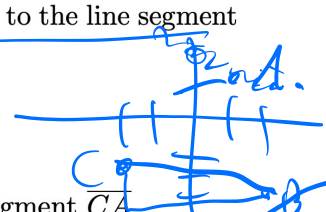
$y-1=-\frac{1}{2}(x+$

$\boxed{y=-\frac{1}{2}x+\frac{3}{2}}$

2. Let A denote the point $(1, 1)$, B denote the point $(2, -3)$, and C denote the point $(-2, -1)$. Use this information in parts (a) and (b).

   (a) Find the slope-intercept form of the line that passes through A and is parallel to the line segment $\overline{BC}$.

$$m=\frac{-3-(-1)}{2-(-2)}=\frac{-2}{4}=-\frac{1}{2}.$$



   (b) Find an equation of the line through B that is perpendicular to the line segment $\overline{CA}$.

$$m=\frac{1-(-1)}{1-(-2)}=\frac{2}{3},\quad m_{\perp}=-\frac{3}{2},\quad y-(-3)=-\frac{3}{2}(x-2)$$

$\boxed{y=-\frac{3}{2}x}$

3. The equation $5x^2 + 60x + 5y^2 + 50y + 125 = 0$ graphs a circle in the xy-plane. Use this equation to answer parts (a)-(e).

   (a) Find the center and radius of the circle given by the equation. $C:(-6,-5)$ $R=6$.

   (b) Find an equation of the form $x = g(y)$ to describe the right semi-circle of the circle with the given equation.

$$x=-6+\sqrt{6^2-(y+5)^2}$$

   (c) Find an equation of the form $y = f(x)$ to describe the bottom semi-circle of the circle with the given equation.

   (d) Find an equation of the form $x = g(y)$ to describe the left semi-circle of the circle with the given equation.

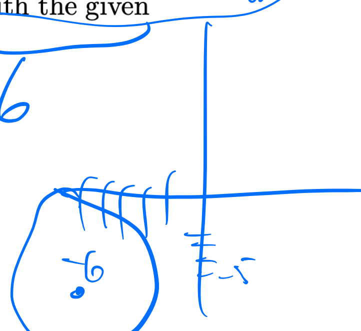
$$y=-5-\sqrt{6^2-(x+6)^2}$$

$$y=-5+\sqrt{6^2-(x+6)^2}$$

   (e) Find an equation of the form $y = f(x)$ to describe the top semi-circle of the circle with the given equation.

$$x^2+12x+36+y^2+10y+\quad+25=0+36$$

$$(x+6)^2+(y+5)^2=6^2.$$



4. Solve for t in the equation $|-2t+1| = 10$.

$-2t+1=10$ or $-2t+1=-10$

$t=\frac{-9}{2}$. $t=\frac{11}{2}$.

5. Solve the inequality $|1+2x| < 4$. Use interval notation to express the solution.

-4 4

$1+2x<4$ & $1+2x>-4$

$x<\frac{3}{2}$ & $x>\frac{-5}{2}$.

$\boxed{\left(-\frac{5}{2}, \frac{3}{2}\right)}$

6. Solve the inequality $|1+3x| > 1$. Use interval notation to express the solution.

-1

$1+3x>1$ OR $1+3x<-1$

$x>0$ OR $x<\frac{-2}{3}$

$\boxed{(-\infty, -\frac{2}{3}) \cup (0, \infty)}$

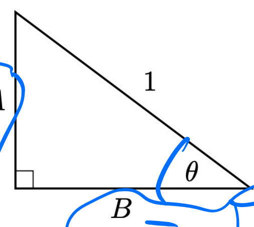
7. Let θ be the angle defined by

$$\theta = \sin^{-1}(23x),$$

where x be a positive number such that $\sin^{-1}(23x)$ is defined. Use this information and the right triangle sketched below to answer parts (a)-(c).

$\sin\theta = 23x = \frac{opp}{Hyp.}$

$23x = A$



$= \sqrt{1-23^2x^2}$

(a) Express A and B in terms of x. *Do not use any trigonometric or inverse trigonometric functions in your answers. Give exact values – do not round.*

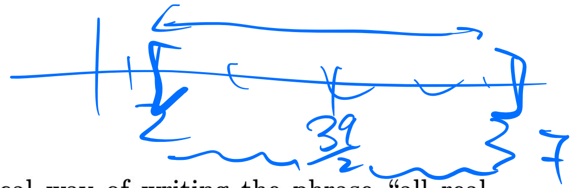
(b) Express the area of the triangle as a function of x. *Do not use any trigonometric or inverse trigonometric functions in your answers. Give exact values – do not round.*

$\frac{1}{2} 23x \cdot \sqrt{1-23^2x^2}$

(c) Express $\sin(2\theta)$ in terms of x. *Do not use any trigonometric or inverse trigonometric functions in your answers. Give exact values – do not round.*

$\sin(2\theta) = 2\sin\theta\cos\theta = 2 \cdot 23x \cdot \sqrt{1-23^2x^2}$.

8. Let $I = [2, 37]$. Use the interval I to answer parts (a)-(c).



(a) Find the length of I. $= 35$

(b) Find the midpoint of I. $= \frac{39}{2}$.

(c) Recall that the expression $|x - a| \leq b$ is a mathematical way of writing the phrase "all real numbers x whose distance from the number a is less than or equal to the real number b." We can use the mathematical expression $|x - a| \leq b$ to describe intervals of the form $[c, d]$. For example, the interval $[-2, 4]$ can be described using

$$|x - 1| \leq 3.$$

Find a and b so that the interval $I = [2, 37]$ can be expressed using the expression $|x - a| \leq b$.

$$\left|x - \frac{39}{2}\right| \leq \frac{35}{2}$$

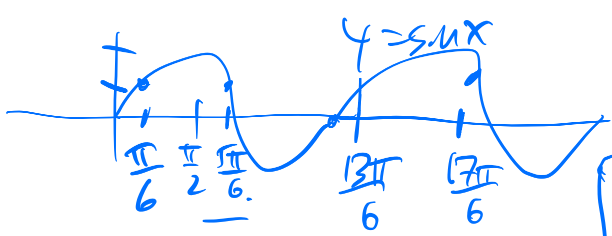
$\sin 2\theta = 2 \cos\theta \sin\theta$

9. Find all solutions in the interval $[0, 12\pi]$ to the equation

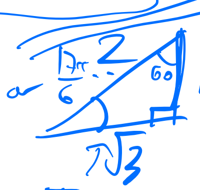
$$\frac{1}{2}\sin\left(\frac{x}{3}\right) = \sin(x/6)\cos(x/6) = \frac{1}{4}. \Rightarrow \boxed{\sin\left(\frac{x}{3}\right) = \frac{1}{2}}$$

$x = \frac{\pi}{6} + 2\pi n$ or $\frac{5\pi}{6} + 2\pi n$

$y = \sin x$



$\frac{x}{3} = \frac{\pi}{6}$, or $\frac{5\pi}{6}$ or $\frac{13\pi}{6}$ or $\frac{17\pi}{6}$



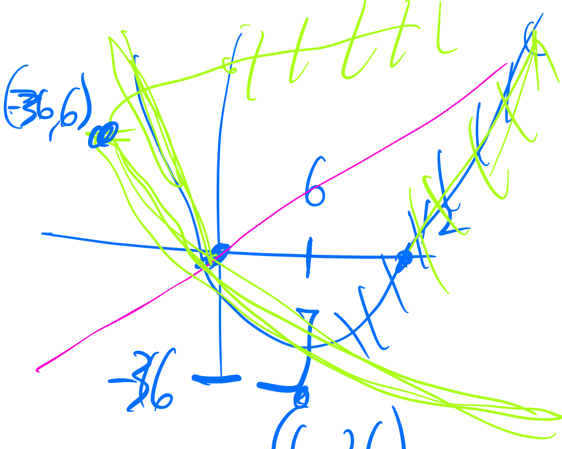
$$\boxed{x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{13\pi}{2}, \frac{17\pi}{2}}$$

10. Let $f(x) = x^2 - 12x$ with domain $x \leq 6$. Use $f(x)$ to answer parts (a) and (b).

(a) Find a formula for f^{-1}.

(b) Find the domain for f^{-1}. Use interval notation to describe the domain.

$y = x(x-12) = x^2 - 12x$



$x = y^2 - 12y$, $y^2 - 12y - x = 0$.

$$y = \frac{12 \pm \sqrt{144 + 4 \cdot 1 \cdot x}}{2 \cdot 1}$$

a) $f^{-1}(x) = \dfrac{12 - \sqrt{144 + 4x}}{2}$

b) Domain of f^{-1}: $[-36, \infty)$.

11. Fill in the blanks. If a numerical value is used, be sure it is an exact value – no decimal approximations are accepted. If a mathematical expression is used, be sure it is as simplified as possible.

(a) $\sin^{-1}(\sin(21\pi/4)) = \boxed{-\pi/4}$.

(b) $\log_{35} \boxed{35} = 1$.

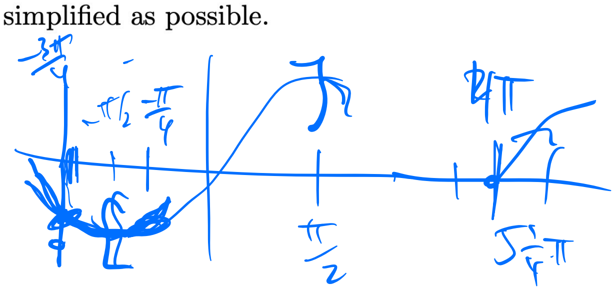
(c) $\cos^{-1}(\cos(23\pi/3)) = \boxed{\pi/3}$.

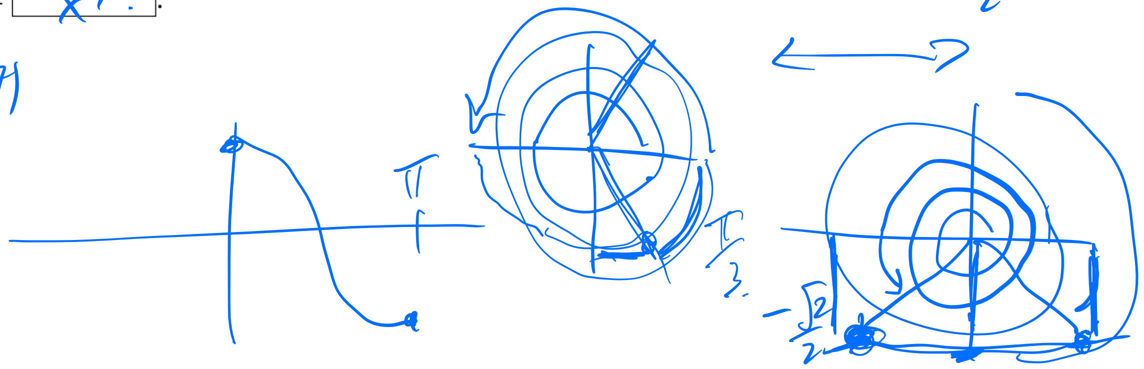
(d) $\sqrt{(2x+98)^2} = \boxed{|2x+98|}$.

(e) $e^{7\ln(x)} = \boxed{x^7}$.

$\log_b 1 = 0.$

$e^{\ln(x^7)}$



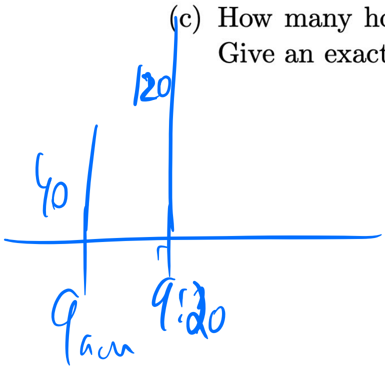


12. A bacteria colony is given to your biology lab group by the lab TA. She tells you that the population of the colony is 40 bacteria when she hands the sample to your group at 9 a.m. In 20 minutes, your group determines that the bacteria population has tripled to 120 bacteria. Assume that the population continues to triple every 20 minutes. Use this information to answer parts (a)-(c).

(a) If $P(t)$ is a function that gives the number of bacteria present in the colonly t hours after 9 a.m., what values of t gives $P(t) = 120$? $t = 1/3$

(b) Find a function $P(t)$ that gives the number of bacteria present in the colony t hours after 9 a.m. (This means $P(0)$ will give the population of the colony at 9 a.m., $P(1)$ will give the population of the colony at 10 a.m., etc.)

(c) How many hours past 9 a.m. will it take for the bacteria population to reach 10001 bacteria? Give an exact value – no decimal approximations accepted. $3t = \log_3 \frac{10001}{40}$ $\boxed{t = \frac{1}{3}\log_3 \frac{10001}{40}}$



$$\boxed{P(t) = 40 \cdot 3^{3t}}$$

$= 120$, $3^{3t} = 3$, $3t = 1$, $t = 1/3$

$40 \cdot 3^{3t} = 10001$ $\quad 3^{3t} = \frac{10001}{40}$

13. Fill in the blanks in the statement below with either a, b, x, $f(x)$, D, R, 1, or $1/f(x)$. NOTE: (1) you may need to use some of the possibilities more than once or not at all when filling in the blanks. (2) The notation $x \in A$ means "the number x is an element in the set A."

Let $f(x)$ be a one-to-one function with domain D and range R. Then $f(x)$ has an inverse function $f^{-1}(x)$ that satisfies

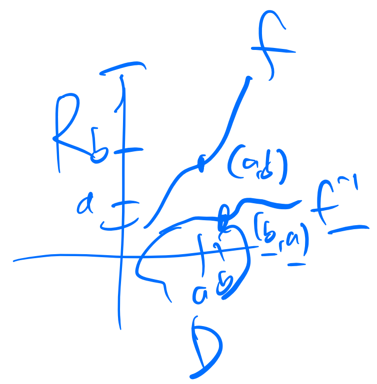
$$f^{-1}(b) = a \text{ if and only if } f(\boxed{a}) = \boxed{b}$$

for all $a \in \boxed{D}$ and $b \in \boxed{R}$. This means that

$$(f \circ f^{-1})(x) = \boxed{x} \text{ for all } x \in \boxed{R}$$

and

$$(f^{-1} \circ f)(x) = \boxed{x} \text{ for all } x \in \boxed{D}.$$



14. Suppose $f(x)$ is a one-to-one function with $f(18) = 194$. What is $f^{-1}(194)$? 18

15. The function $f(x) = \dfrac{50x + 13}{x - 71}$ is a one-to-one function on its domain. Use $f(x)$ to answer parts (a)-(c).

$x \neq 71$

(a) Find the domain of f and write the domain using interval notation. $(-\infty, 71) \cup (71, \infty)$.

(b) Find the range of f and write the range using interval notation. $(-\infty, 50) \cup (50, \infty)$.

(c) Find a formula for $f^{-1}(x)$.

$$y = \frac{50x + 13}{x - 71}, \quad x = \frac{50y + 13}{y - 71}, \quad x \cdot y - x \cdot 71 = 50y + 13$$

$$(x - 50)y = 71x + 13$$

$$f^{-1}(x) = \frac{71x + 13}{x - 50}$$

16. (a) Find a function $f(x)$ such that the method

$$\frac{\sqrt{9x+6} - \sqrt{78}}{\sqrt{7x+19} - 5\sqrt{3}} = \frac{(\sqrt{9x+6} - \sqrt{78}) \cdot f(x)}{(\sqrt{7x+19} - 5\sqrt{3}) \cdot f(x)}$$

makes it possible to evaluate

$$\lim_{x \to 8} \frac{\sqrt{9x+6} - \sqrt{78}}{\sqrt{7x+19} - 5\sqrt{3}}.$$

$f(x) =$

(b) Evaluate $\lim_{x \to 8} \dfrac{\sqrt{9x+6} - \sqrt{78}}{\sqrt{7x+19} - 5\sqrt{3}}$. Be sure to show all steps and use correct mathematical notation.

$$\lim_{x \to 8} \frac{(\sqrt{9x+6} - \sqrt{78})(\sqrt{9x+6} + \sqrt{78})(\sqrt{7x+19} + \sqrt{75})}{(\sqrt{7x+19} - \sqrt{75})(\sqrt{9x+6} + \sqrt{78})(\sqrt{7x+19} + \sqrt{75})}$$

$$= \lim_{x \to 8} \frac{9x + 6 - 78}{7x + 19 - 75} \cdot \frac{\sqrt{7x+19} + \sqrt{75}}{\sqrt{9x+6} + \sqrt{78}} = \frac{9}{7} \cdot \frac{\cancel{2}\sqrt{75}}{\cancel{2} \cdot \sqrt{78}}$$

$$= \lim_{x \to 8} \frac{9\cancel{(x-8)}}{7\cancel{(x-8)}} \cdot$$

17. Let $g(x) = \dfrac{\tan(20x)}{|33x|}$. Use this definition of $g(x)$ in parts (a)-(h).

$\to 1$ $\quad$ 20/33

(a) For small positive values of x, we can rewrite $g(x)$ without the use of the absolute value sign. If $x > 0$ and small, what is an expression for $g(x)$ that does not have an absolute value sign in the expression?

$\tan(20x)/33x. = \dfrac{\sin(20x)}{20x} \cdot \dfrac{20x}{\cos(20x)} \cdot \dfrac{1}{33x}$ $\to 1$

(b) To compute $\lim_{x\to 0+} g(x)$, we must multiply by a "special 1" of the form $\dfrac{f(x)}{f(x)}$. What function $f(x)$ should we use to build the special 1?

$\boxed{f = 20x}$ (or $f = 20$.)

(c) Compute $\lim_{x\to 0+} g(x)$, if possible. If the limit does not exist, explain why it does not. Be sure to provide a mathematical justification of your conclusion. 20/33.

(d) If $x < 0$ with $|x|$ small, we can rewrite $g(x)$ without the use of the absolute value sign. If $x < 0$ with $|x|$ small, what is an expression for $g(x)$ that does not have an absolute value sign in the expression?

$\tan(20x)/(-33x)$.

(e) To compute $\lim_{x\to 0-} g(x)$, we must multiply by a "special 1" of the form $\dfrac{f(x)}{f(x)}$. What function $f(x)$ should we use to build the special 1?

$(f = 20x)$

(f) Compute $\lim_{x\to 0-} g(x)$, if possible. If the limit does not exist, explain why it does not. Be sure to provide a mathematical justification of your conclusion. $-20/33$, same.

(g) Compute $\lim_{x\to 0} g(x)$, if possible. If the limit does not exist, explain why it does not. Be sure to provide a mathematical justification of your conclusion. DNE $\lim_{x\to 0^+} \neq \lim_{x\to 0^-}$

(h) Can you define $g(0)$ in a way that extends $g(x) = \dfrac{\tan(20x)}{|33x|}$ to be continuous at $x = 0$? Be sure to provide a mathematical justification of your conclusion.

For continuous extension to exist, need $\lim_{x\to 0}$ to exist. It doesn't so: No.

18. Find the limit, if it exists. If the limit does not exist, explain it does not. Be sure to show all work and use correct mathematical notation – it will be graded.

(a) $\displaystyle\lim_{x\to 0} \frac{x\sin(5x)}{1-\cos(6x)}$ (b) $\displaystyle\lim_{x\to 4} \frac{2-\sqrt{x}}{3x^2-11x-4}$ (c) $\displaystyle\lim_{x\to\infty} e^{3/x}\cos\left(\frac{4}{x}\right)$ $= \displaystyle\lim_{t\to 0^+} e^{3t}\cos(4t)$

$t = \frac{1}{x}$

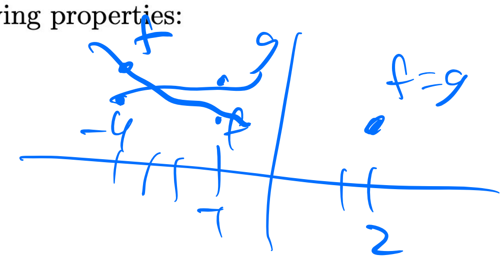
$= 1.$

$\dfrac{x\sin(5x)\cdot 5x\,(1+\cos6x)}{5x(1-\cos(6x))(1+\cos6x)}$

$= \dfrac{\sin5x}{5x}\cdot\dfrac{(6x)^2}{\sin^2 6x}\cdot\dfrac{5x^2(1+\cos6x)}{36x^2} = \dfrac{5}{18}.$

19. Suppose $f(x)$ and $g(x)$ are two functions that have the following properties:

- The domain of f is $(-\infty, \infty)$.
- The domain of g is $(-\infty, \infty)$.
- $f(-4) > g(-4)$.
- $f(-1) < g(-1)$.
- $x = 2$ is the only solution to the equation $f(x) = g(x)$.



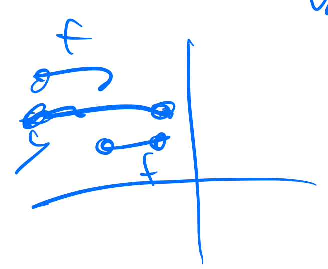
Determine if the following statements are True or False. If a statement is False, provide an example of functions f and g that satisfy the above conditions but make the statement false.

(a) Either $f(x)$ or $g(x)$ is not continuous on $[-4, -1]$. TRUE

(b) $\displaystyle\lim_{x\to 2}[f(x) - g(x)] = 0$. FALSE, limits not known to agree w/ value

(c) $f(c) - g(c) = 0$ for some number $-4 < c < -1$.

FALSE could be not continuous.



20. Let $f(x)$ and $g(x)$ be defined as

$$f(x) = \begin{cases} x^2 - x + 3 & \text{if } x < 3 \\ 2x^2 + 3x - 7 & \text{if } x > 3 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} x^2 + x + 1 & \text{if } x < 1 \\ 2x^2 + 7x + 5 & \text{if } x > 1 \end{cases}$$

$\to 9$, $\to 20$ $\qquad$ $\to 3$, $\to 14.$

Now define $h(x)$ as

$$h(x) = f(x) - g(x-2).$$

Use the above definitions of f, g, and h in parts (a)-(d).

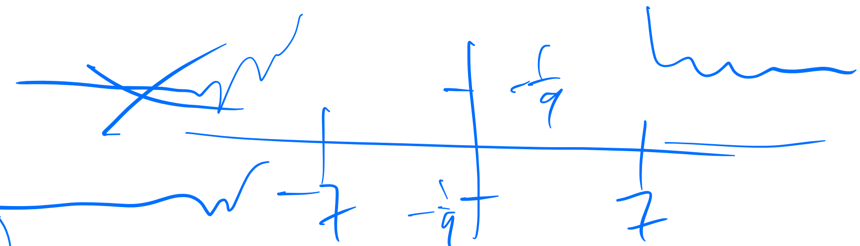
(a) Compute $\lim_{x\to 3} f(x)$, if possible. If it is not possible, explain why it is not possible. DNE

(b) Compute $\lim_{x\to 1} g(x)$, if possible. If it is not possible, explain why it is not possible. DNE

(c) Compute $\lim_{x\to 3} h(x)$, if possible. If it is not possible, explain why it is not possible. $=6.$

(d) Is it possible to define $h(3)$ in such a way that $h(x)$ is continuous at $x = 3$? If so, what should $h(3)$ be? If not, why is it not possible?

Yes $h(3)=6.$



21. Let $f(x) = \dfrac{\sqrt{49 - x^2}}{9x - 9}$ and $g(x) = \dfrac{\sqrt{x^2 - 49}}{9x - 9}$. Use $f(x)$ and $g(x)$ in parts (a)-(h).

(a) Use interval notation to describe the domain of f. $x \neq 1$, $|x| \le 7$, $[-7,1)\cup(1,7]$

(b) Use interval notation to describe the domain of g. $|x| \ge 7$ $(-\infty,-7]\cup[7,\infty).$

(c) Find the **equation(s)** of the horizontal asymptote(s) for the graph of $y = f(x)$. None.

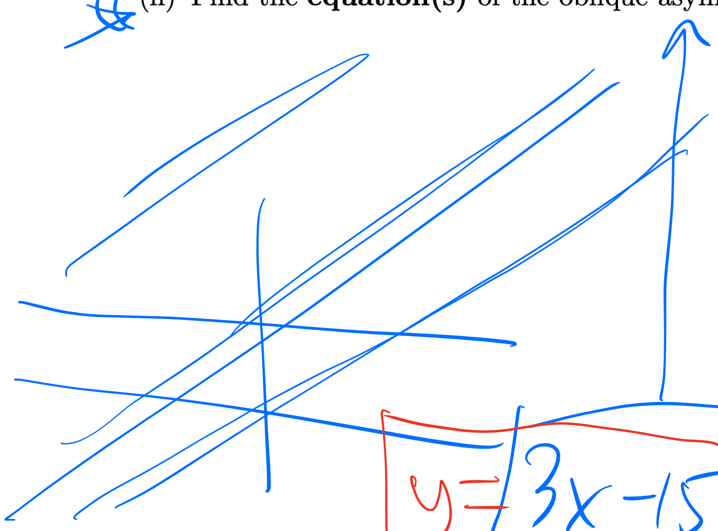
(d) Find the **equation(s)** of the horizontal asymptote(s) for the graph of $y = g(x)$. $\to y=\frac{1}{9},\ y=-\frac{1}{9}$

(e) Find the **equation(s)** of the vertical asymptote(s) for the graph of $y = f(x)$.

(f) Find the **equation(s)** of the vertical asymptote(s) for the graph of $y = g(x)$. $x=1.$

(g) Find the **equation(s)** of the oblique asymptote(s) for the graph of $y = f(x)$. None

(h) Find the **equation(s)** of the oblique asymptote(s) for the graph of $y = g(x)$. None.



$\boxed{y = 3x - 15}$

$$\frac{3x^2+17}{x+5}$$

$$\begin{array}{r} 3x - 15 + \frac{92}{x+5} \\ (x+5)\overline{)\,3x^2 + 0x + 17} \\ -(3x^2 + 15x) \quad \\ -15x + 17 \\ +15x + 75 \\ \hline 92 \end{array}$$

22. Suppose that

$$f(x) = \begin{cases} 4x^2 + 9 & \text{if } x \le -3 \\ ax + b & \text{if } x > -3 \end{cases},$$

where a and b are some constants. Use $f(x)$ in parts (a)-(c).

(a) Compute $\lim_{x \to -3^-} f(x)$. $x(-3)+6$.

(b) Compute $\lim_{x \to -3^+} f(x)$. $\rightarrow$ 45.

(c) If $f(x)$ is to be continuous at $x = -3$, what expression must b equal? $b = +3$.

(d) If $f(x)$ is not continuous at $x = -3$, what type of discontinuity will f have at $x = -3$? Jump discontinuity.